EPICENTER Spring Course in Epistemic Game Theory

Maastricht University, July 5–19, 2016

Exam

July 19, 14.00–17.00, Room H 0.06

Problem 1: Choosing the middle number (40 points)

Barbara, Chris and you have invented a new card game. The rules are simple: Barbara gets the cards with numbers 1 and 4, you get the cards with numbers 2 and 5, and Chris gets the cards with numbers 3 and 6. You must all simultaneously put one of the two cards on the table. The person whose card number is the *middle* of the three numbers on the table *wins*, and the other two *loose*. The two losers must pay the number of their card in euros. The winner does not need to pay anything, but also gets nothing in return. Hence, in this game you can only *lose* money.

(a) (4 points) What are the rational cards for you, Barbara and Chris? For every rational card, find a belief about the cards of the other players for which it is optimal. For every irrational card, if any, explain why it is strictly dominated by the other card.

(b) (5 points) Use the associated algorithm to find the cards that you, Barbara and Chris can rationally choose under 1-fold belief in rationality, the cards that can rationally be chosen under expressing up to 2-fold belief in rationality, and the cards that can rationally be chosen under common belief in rationality. Which algorithm do you use? Whom do you expect to win under common belief in rationality?

(c) (3 points) Explain verbally and formally what it means for a type to have a *simple belief hierarchy.*

(d) (3 points) Explain the difference between the concepts of *common belief in rationality* and *Nash equilibrium.*

(e) (3 points) Discuss the conceptual problems you see with the concept of *Nash equilibrium.*

After playing for a while, you introduce a new rule: If the sum of the card numbers on the table is 15, then Chris will win. Hence, in that case, Chris pays nothing whereas Barbara and you pay the number of the chosen card. In all other cases, the rules stay the same as before.

(f) (4 points) What are the rational cards for you, Barbara and Chris? For every rational card, find a belief about the cards of the other players for which it is optimal. For every irrational card, if any, explain why it is strictly dominated by the other card.

(g) (3 points) Which cards can rationally be chosen by you, Barbara and Chris under common belief in rationality? Explain your answer.

(h) (5 points) Construct an epistemic model such that for every card in (g) there is a type for the corresponding player that expresses *common belief in rationality*, and for which the associated card is *optimal*.

(i) (3 points) Which of the types in your epistemic model, if any, have a *simple belief hierarchy*, and which have not? Explain your answer.

(j) (7 points) Show that under *common belief in rationality* with a *simple belief hierarchy*, you cannot rationally choose the card with number 5.

Problem 2: Even or odd (30 points)

After the card game from Problem 1, Barbara and you decide to continue with yet another game. You and Barbara must simultaneously write down, on a piece of paper, some number from $\{0, 1, 2, 3\}$. If the sum of the two numbers is even and less than 4, you receive your own number in euros and Barbara gets nothing. (Here, we count 0 as an even number). If the sum of the two numbers is odd and less than 4, Barbara receives her own number in euros and you get nothing. If the sum of the two numbers is at least 4, you and Barbara both get 4 euros.

(a) (3 points) Model this situation as a game between Barbara and you. That is, list the possible choices for you and Barbara, and make a table with the utilities for you and Barbara for every pair of choices.

(b) (3 points) Determine the numbers that you and Barbara can rationally choose under *common belief in rationality*. Motivate your answer.

(c) (5 points) Use the associated algorithm to determine the numbers that you and Barbara can rationally choose under *common full in belief in "caution and primary belief in rationality"*. Which algorithm do you use?

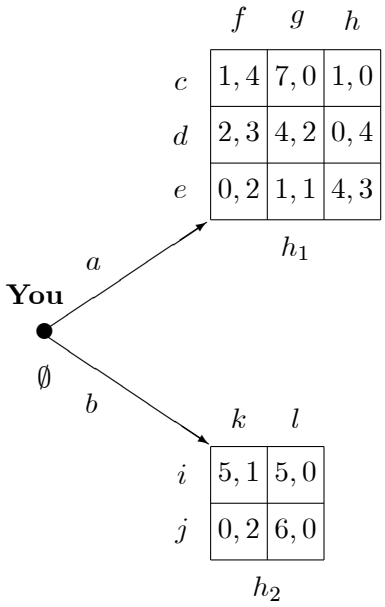
(d) (5 points) Use the associated algorithm to determine the numbers that you and Barbara can rationally choose under *common assumption of rationality*. Which algorithm do you use?

(e) (7 points) Use the associated algorithm to determine the numbers that you and Barbara can rationally choose under *common full in belief in "caution and respect of preferences"*. Which algorithm do you use?

(f) (7 points) Construct an epistemic model such that for every number in (e) there is a cautious type that expresses *common full in belief in "caution and respect of preferences"* and for which that number is optimal.

Problem 3: A dynamic game (30 points)

Consider the following dynamic game between Barbara and you. At the information sets h_1 and h_2, your choices correspond to the rows and Barbara's choices correspond to the columns.



(a) (5 points) Explain intuitively the difference between the concepts of *common belief in future rationality* and *common strong belief in rationality.*

(b) (10 points) Use the associated algorithm to find the strategies that you and Barbara can rationally choose under *common belief in future rationality.* Which algorithm do you use?

(c) (10 points) Use the associated algorithm to find the strategies that you and Barbara can rationally choose under *common strong belief in rationality.* Which algorithm do you use?

(d) (5 points) Explain intuitively why both concepts lead to different strategies for Barbara.