EPICENTER Summer Course on Epistemic Game Theory

Incomplete Information, Unawareness and Psychological Games

Day 2: *Common Belief in Rationality in Standard Games*

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Introduction

- In a game, the outcome depends on the choices of your opponents.
- States: choice-combinations by your opponents.
- Every player faces his own decision problem.
- To make a good decision, you must reason about decision problems of others.
- Central reasoning concept: Common belief in rationality.
- How to formalize common belief in rationality?
- How to characterize choices that are possible under common belief in rationality?
- Is common belief in rationality always possible?

Outline

- Games as decision problems
- Belief hierarchies, beliefs diagrams and types
- Common belief in rationality
- Recursive elimination procedure
- Possibility of common belief in rationality

Games as Decision Problems

Example: Going to a Party

You	*blue*	*green*	*red*	*yellow*
blue	0	4	4	4
green	3	0	3	3
red	2	2	0	2
yellow	1	1	1	0

Barbara	*blue*	*green*	*red*	*yellow*
blue	0	2	2	2
green	1	0	1	1
red	4	4	0	4
yellow	3	3	3	0

Story

- This evening, you are going to a party together with your friend Barbara.
- You must both decide which color to wear: *blue*, *green*, *red* or *yellow*.
- You dislike wearing the same color as Barbara, and the same holds for Barbara.

Games as Decision Problems

Example: Going to a Party

You	*blue*	*green*	*red*	*yellow*
blue	0	4	4	4
green	3	0	3	3
red	2	2	0	2
yellow	1	1	1	0

Barbara	*blue*	*green*	*red*	*yellow*
blue	0	2	2	2
green	1	0	1	1
red	4	4	0	4
yellow	3	3	3	0

- Your choice *yellow* is irrational: It is strictly dominated by $(0.5)\cdot$ *blue* $+$ $(0.5)\cdot$ *green*.
- Similarly, Barbara's choice *green* is irrational.
- If you believe in Barbara's rationality, you must assign probability 0 to her choice *green*.
- Eliminate state *green* from your decision problem.
- Then, your choice *red* becomes irrational.

Definition (Standard game)

A standard game specifies

(a) a finite set of players I,

(b) for every player i a finite set of choices C_i,

(c) for every player i a decision problem $(C_i, C_{-i}, \succsim_i)$.

- We assume: conditional preference relation $\succsim_i$ has an expected utility representation u_i.
- It is important to reason about the decision problems of others.

- Idea of common belief in rationality:

  you believe that your opponents choose rationally,

  you believe that your opponents believe that the other players choose rationally,

  and so on.

- It puts restrictions on

  your belief about the opponents' choices

  (first-order belief),

  your belief about the opponents' first-order beliefs
  (second-order belief),

  your belief about the opponents' second-order beliefs
  (third-order belief),

  and so on.

Definition (Belief hierarchy)

A belief hierarchy for player i specifies

a first-order belief: a belief about the opponents' choice combinations,

a second-order belief: a belief about the opponents' first-order beliefs,

a third-order belief: a belief about the opponents' second-order beliefs,

and so on, ad infinitum.

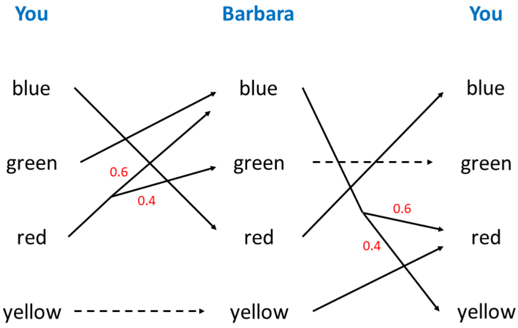
- Problem: Belief hierarchy is an infinite object.
- We will see: It has finite representation in terms of

  beliefs diagram and

  epistemic model with types.

You	*blue*	*green*	*red*	*yellow*
blue	0	4	4	4
green	3	0	3	3
red	2	2	0	2
yellow	1	1	1	0

Barbara	*blue*	*green*	*red*	*yellow*
blue	0	2	2	2
green	1	0	1	1
red	4	4	0	4
yellow	3	3	3	0



Belief Hierarchies, Beliefs Diagrams and Types

Types

- A beliefs diagram can be used to visualize belief hierarchies.
- Epistemic models with types provide a finite mathematical encoding of belief hierarchies.
- Recall: a belief hierarchy specifies

  a first-order belief: a belief about the opponents' choice combinations,

  a second-order belief: a belief about the opponents' first-order beliefs,

  a third-order belief: a belief about the opponents' second-order beliefs,

  and so on, ad infinitum.
- Hence, a belief hierarchy specifies a belief about

  the opponents' choices, and

  the opponents' belief hierarchies.
- Following Harsanyi (1967-1968), we call a belief hierarchy a type.

Definition (Epistemic model)

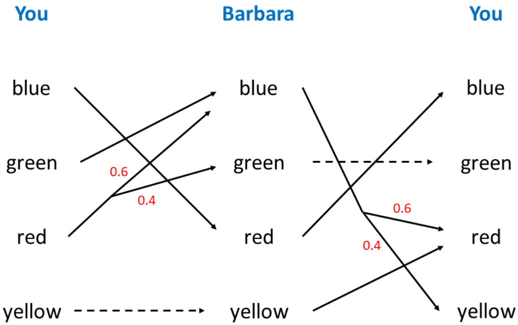
An epistemic model specifies for every player i

a finite set T_i of possible types, and

for every type $t_i \in T_i$ a belief $b_i(t_i) \in \Delta(C_{-i} \times T_{-i})$ about the opponents' choice-type combinations.

- Based on Harsanyi (1967-1968).
- For every type we can derive a full belief hierarchy.
- It thus provides a finite mathematical encoding of belief hierarchies.

Types	$T_1 = \{t_1^{blue}, t_1^{green}, t_1^{red}, t_1^{yellow}\}$ $T_2 = \{t_2^{blue}, t_2^{green}, t_2^{red}, t_2^{yellow}\}$
Beliefs for player 1	$b_1(t_1^{blue}) = (red,\ t_2^{red})$ $b_1(t_1^{green}) = (blue,\ t_2^{blue})$ $b_1(t_1^{red}) = (0.6) \cdot (blue,\ t_2^{blue}) + (0.4) \cdot (green,\ t_2^{green})$ $b_1(t_1^{yellow}) = (yellow,\ t_2^{yellow})$
Beliefs for player 2	$b_2(t_2^{blue}) = (0.6) \cdot (red,\ t_1^{red}) + (0.4) \cdot (yellow,\ t_1^{yellow})$ $b_2(t_2^{green}) = (green,\ t_1^{green})$ $b_2(t_2^{red}) = (blue,\ t_1^{blue})$ $b_2(t_2^{yellow}) = (red,\ t_1^{red})$



Beliefs for player 1	$b_1(t_1^{blue})$	$=$	$(red,\ t_2^{red})$
	$b_1(t_1^{green})$	$=$	$(blue,\ t_2^{blue})$
	$b_1(t_1^{red})$	$=$	$(0.6) \cdot (blue,\ t_2^{blue}) + (0.4) \cdot (green,\ t_2^{green}$
	$b_1(t_1^{yellow})$	$=$	$(yellow,\ t_2^{yellow})$
Beliefs for player 2	$b_2(t_2^{blue})$	$=$	$(0.6) \cdot (red,\ t_1^{red}) + (0.4) \cdot (yellow,\ t_1^{yellow}$
	$b_2(t_2^{green})$	$=$	$(green,\ t_1^{green})$
	$b_2(t_2^{red})$	$=$	$(blue,\ t_1^{blue})$
	$b_2(t_2^{yellow})$	$=$	$(red,\ t_1^{red})$

Definition (Epistemic model)

An epistemic model specifies for every player i

a finite set T_i of possible types, and

for every type $t_i \in T_i$ a belief $b_i(t_i) \in \Delta(C_{-i} \times T_{-i})$ about the opponents' choice-type combinations.

- Belief hierarchies can also be encoded by Kripke-structures (Kripke, 1963) and Aumann-structures (Aumann, 1974, 1976).
- In Bach and Perea (2023) it is shown how to go from epistemic models with types to Aumann-structures, and *vice versa*, while preserving the belief hierarchy.

- Consider a type t_i, and its first-order belief $b_i^1(t_i)$ about the opponents' choices.
- Choice c_i is optimal for type t_i if

$$u_i(c_i, b_i^1(t_i)) \geq u_i(c_i', b_i^1(t_i)) \text{ for all choices } c_i' \in C_i.$$

Definition (Belief in opponents' rationality)

Type t_i believes in the opponents' rationality if

$b_i(t_i)$ only assigns positive probability to pairs (c_j, t_j) where

c_j is optimal for t_j.

Common Belief in Rationality

Definition

Definition (Common belief in rationality)

Type t_i expresses 1-fold belief in rationality if t_i believes in the opponents' rationality.

Type t_i expresses 2-fold belief in rationality if t_i only assigns positive probability to opponents' types that express 1-fold belief in rationality.

Type t_i expresses 3-fold belief in rationality if t_i only assigns positive probability to opponents' types that express 2-fold belief in rationality.

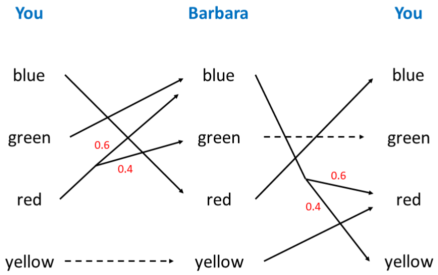
And so on.

Type t_i expresses common belief in rationality if t_i expresses k-fold belief in rationality for all k.

- Based on Tan and Werlang (1988).

- In the literature, this concept is also known as correlated rationalizability (Brandenburger and Dekel (1987)).
- Rationalizability (Bernheim (1984), Pearce (1984)) is obtained if in games with three players or more we impose the following additional condition:
- Player i's belief about opponent j's choice must be independent from his belief about opponent k's choice.
- Verbal formulations appear in Friedell (1969) and Spohn (1982).
- Aumann (1987) uses structural rationality within an Aumann-model:

  at every state, all players choose optimally given their beliefs.
- It implies common belief in rationality.



- Types t_1^{red} and t_2^{blue} do not express 1-fold belief in rationality.
- Types t_1^{green} and t_2^{yellow} do not express 2-fold belief in rationality.
- Types t_1^{yellow} and type t_2^{green} do not express 3-fold belief in rationality.
- Types t_1^{blue} and type t_2^{red} express common belief in rationality.

Theorem (Sufficient condition for common belief in rationality)

Consider an epistemic model in which all types believe in the opponents' rationality.

Then, all types in the epistemic model express common belief in rationality.

- Proof: Show that every type expresses k-fold belief in rationality, for all k.
- Every type expresses 1-fold belief in rationality.
- Since a type can only assign positive probability to other types in the same model, every type expresses 2-fold belief in rationality.
- But then, every type also expresses 3-fold belief in rationality.
- And so on.
- Hence, all types express common belief in rationality. ∎

Definition

Player i can rationally make choice c_i under common belief in rationality if there is some epistemic model, and some type t_i within it, such that

type t_i expresses common belief in rationality, and

choice c_i is optimal for type t_i.

- Find a recursive elimination procedure that yields precisely those choices that can rationally be made under common belief in rationality.

Recursive Elimination Procedure

- Which choices for player *i* are optimal for some belief?
- By strict dominance theorem from Day 1, these are exactly the choices that are

  not strictly dominated.
- Eliminate, for every player *i*, those choices in his decision problem that

  are strictly dominated.
- 1-fold reduced decision problem.

Recursive Elimination Procedure

Round 2

- Which choices are optimal for player i if he expresses 1-fold belief in rationality?
- Player i assigns probability zero to opponents' choices that did not survive round 1.
- Eliminate states in i's decision problem that involve opponents' choices which did not survive round 1.
- Reduced decision problem.
- Within reduced decision problem,

  the choices that are optimal for some belief are

  the choices that are not strictly dominated.
- From reduced decision problem, eliminate all strictly dominated choices.
- 2-fold reduced decision problem.

Recursive Elimination Procedure

Round 3

- Which choices are optimal for player i if he expresses up to 2-fold belief in rationality?
- Player i assigns probability zero to opponents' choices that did not survive round 2.
- Eliminate states in i's decision problem that involve opponents' choices which did not survive round 2.
- Reduced decision problem.
- Within reduced decision problem,

  the choices that are optimal for some belief are

  the choices that are not strictly dominated.
- From reduced decision problem, eliminate all strictly dominated choices.
- 3-fold reduced decision problem.

Definition (Iterated elimination of strictly dominated choices)

Set up decision problems for all players.

Round 1. From every decision problem, eliminate strictly dominated choices.

1-fold reduced decision problems

Round 2. From every 1-fold reduced decision problem, eliminate states that involve opponents' choices that did not survive round 1.

Within reduced decision problem so obtained, eliminate all strictly dominated choices.

2-fold reduced decision problems

Continue until no further states and choices can be eliminated.

- For every player there is at least one choice that survives procedure.

Theorem (Characterization)

(a) The choices that are optimal for a type that expresses up to k-fold belief in rationality

are exactly those choices that survive $(k+1)$-fold elimination of strictly dominated choices.

(b) The choices that can rationally be made under common belief in rationality

are exactly those choices that survive iterated elimination of strictly dominated choices.

- Based on Tan and Werlang (1988).
- Adam Brandenburger (2014) calls it the Fundamental Theorem of Epistemic Game Theory.
- For every player there is at least one type that expresses common belief in rationality.

Recursive Elimination Procedure

Example: Going to a Party

You	*blue*	*green*	*red*	*yellow*
blue	0	4	4	4
green	3	0	3	3
red	2	2	0	2
yellow	1	1	1	0

Barbara	*blue*	*green*	*red*	*yellow*
blue	0	2	2	2
green	1	0	1	1
red	4	4	0	4
yellow	3	3	3	0

Round 1: For you, *yellow* is strictly dominated by $(0.5) \cdot$ *blue* + $(0.5) \cdot$ *green*

For Barbara, *green* is strictly dominated by $(0.5) \cdot$ *red* + $(0.5) \cdot$ *yellow*

Recursive Elimination Procedure

Example: Going to a Party

You	*blue*	*green*	*red*	*yellow*
blue	0	4	4	4
green	3	0	3	3
red	2	2	0	2
~~*yellow*~~	~~1~~	~~1~~	~~1~~	~~0~~

Barbara	*blue*	*green*	*red*	*yellow*
blue	0	2	2	2
~~*green*~~	~~1~~	~~0~~	~~1~~	~~1~~
red	4	4	0	4
yellow	3	3	3	0

Round 1: For you, *yellow* is strictly dominated by $(0.5) \cdot$ *blue* + $(0.5) \cdot$ *green*

For Barbara, *green* is strictly dominated by $(0.5) \cdot$ *red* + $(0.5) \cdot$ *yellow*

Recursive Elimination Procedure

Example: Going to a Party

You	*blue*	*green*	*red*	*yellow*
blue	0	4	4	4
green	3	0	3	3
red	2	2	0	2
~~*yellow*~~	~~1~~	~~1~~	~~1~~	~~0~~

Barbara	*blue*	*green*	*red*	*yellow*
blue	0	2	2	2
~~*green*~~	~~1~~	~~0~~	~~1~~	~~1~~
red	4	4	0	4
yellow	3	3	3	0

Round 2: For you, *red* is strictly dominated by *green*

For Barbara, *blue* is strictly dominated by *yellow*

Recursive Elimination Procedure

Example: Going to a Party

You	*blue*	*green*	*red*	*yellow*
blue	0	4	4	4
green	3	0	3	3
~~*red*~~	~~2~~	~~2~~	~~0~~	~~2~~
~~*yellow*~~	~~1~~	~~1~~	~~1~~	~~0~~

Barbara	*blue*	*green*	*red*	*yellow*
~~*blue*~~	~~0~~	~~2~~	~~2~~	~~2~~
~~*green*~~	~~1~~	~~0~~	~~1~~	~~1~~
red	4	4	0	4
yellow	3	3	3	0

Round 2: For you, *red* is strictly dominated by *green*

For Barbara, *blue* is strictly dominated by *yellow*

Recursive Elimination Procedure

Example: Going to a Party

You	*blue*	*green*	*red*	*yellow*
blue	0	4	4	4
green	3	0	3	3
~~*red*~~	~~2~~	~~2~~	~~0~~	~~2~~
~~*yellow*~~	~~1~~	~~1~~	~~1~~	~~0~~

Barbara	*blue*	*green*	*red*	*yellow*
~~*blue*~~	~~0~~	~~2~~	~~2~~	~~2~~
~~*green*~~	~~1~~	~~0~~	~~1~~	~~1~~
red	4	4	0	4
yellow	3	3	3	0

Round 3: For you, *green* is strictly dominated by *blue*

For Barbara, *yellow* is strictly dominated by *red*

Recursive Elimination Procedure

Example: Going to a Party

You	*blue*	*green*	*red*	*yellow*
blue	0	4	4	4
green	3	0	3	3
red	2	2	0	2
yellow	1	1	1	0

Barbara	*blue*	*green*	*red*	*yellow*
blue	0	2	2	2
green	1	0	1	1
red	4	4	0	4
yellow	3	3	3	0

Round 3: For you, *green* is strictly dominated by *blue*

For Barbara, *yellow* is strictly dominated by *red*

Under common belief in rationality,

you can only rationally wear *blue, and*

Barbara can only rationally wear *red.*

Story

- It is now one year later. Chris has heard some good stories about the party last year, and would like to join.
- You have become tired of wearing blue all the time, and have given all blue clothes to charity.
- Due to the latest fashion developments, Barbara's preferences over colors have changed.
- Chris only has blue and yellow clothes in his wardrobe.
- As before, you all dislike it when a friend wears the same color.
- Which colors could you rationally wear under common belief in rationality?

Recursive Elimination Procedure

Example: When Chris Joins the Party

You	(b,b)	(g,b)	(r,b)	(y,b)	(b,y)	(g,y)	(r,y)	(y,y)
green	3	0	3	3	3	0	3	3
red	2	2	0	2	2	2	0	2
yellow	1	1	1	0	0	0	0	0

Barb	(g,b)	(r,b)	(y,b)	(g,y)	(r,y)	(y,y)
blue	0	0	0	3	3	3
green	0	4	4	0	4	4
red	1	0	1	1	0	1
yellow	2	2	0	0	0	0

Chris	(g,b)	(r,b)	(y,b)	(g,g)	(r,g)	(y,g)	(g,r)	(r,r)	(y,r)	(g,y)	(r,y)	(y,y)
blue	0	0	0	2	2	2	2	2	2	2	2	2
yellow	1	1	0	1	1	0	1	1	0	0	0	0

Recursive Elimination Procedure

Example: When Chris Joins the Party

You	(b,b)	(g,b)	(r,b)	(y,b)	(b,y)	(g,y)	(r,y)	(y,y)
green	3	0	3	3	3	0	3	3
red	2	2	0	2	2	2	0	2
yellow	1	1	1	0	0	0	0	0

Barb	(g,b)	(r,b)	(y,b)	(g,y)	(r,y)	(y,y)
blue	0	0	0	3	3	3
green	0	4	4	0	4	4
red	1	0	1	1	0	1
yellow	2	2	0	0	0	0

Chris	(g,b)	(r,b)	(y,b)	(g,g)	(r,g)	(y,g)	(g,r)	(r,r)	(y,r)	(g,y)	(r,y)	(y,y)
blue	0	0	0	2	2	2	2	2	2	2	2	2
yellow	1	1	0	1	1	0	1	1	0	0	0	0

Round 1: For you, *yellow* is strictly dominated by $(0.4) \cdot$ *green* + $(0.6) \cdot$ *red* .

Recursive Elimination Procedure

Example: When Chris Joins the Party

You	(b,b)	(g,b)	(r,b)	(y,b)	(b,y)	(g,y)	(r,y)	(y,y)
green	3	0	3	3	3	0	3	3
red	2	2	0	2	2	2	0	2
~~*yellow*~~	~~1~~	~~1~~	~~1~~	~~0~~	~~0~~	~~0~~	~~0~~	~~0~~

Barb	(g,b)	(r,b)	(y,b)	(g,y)	(r,y)	(y,y)
blue	0	0	0	3	3	3
green	0	4	4	0	4	4
red	1	0	1	1	0	1
yellow	2	2	0	0	0	0

Chris	(g,b)	(r,b)	(y,b)	(g,g)	(r,g)	(y,g)	(g,r)	(r,r)	(y,r)	(g,y)	(r,y)	(y,y)
blue	0	0	0	2	2	2	2	2	2	2	2	2
yellow	1	1	0	1	1	0	1	1	0	0	0	0

Round 1: For you, *yellow* is strictly dominated by $(0.4) \cdot$ *green* + $(0.6) \cdot$ *red* .

Recursive Elimination Procedure

Example: When Chris Joins the Party

You	(b,b)	(g,b)	(r,b)	(y,b)	(b,y)	(g,y)	(r,y)	(y,y)
green	3	0	3	3	3	0	3	3
red	2	2	0	2	2	2	0	2
~~*yellow*~~	~~1~~	~~1~~	~~1~~	~~0~~	~~0~~	~~0~~	~~0~~	~~0~~

Barb	(g,b)	(r,b)	(y,b)	(g,y)	(r,y)	(y,y)
blue	0	0	0	3	3	3
green	0	4	4	0	4	4
red	1	0	1	1	0	1
yellow	2	2	0	0	0	0

Chris	(g,b)	(r,b)	(y,b)	(g,g)	(r,g)	(y,g)	(g,r)	(r,r)	(y,r)	(g,y)	(r,y)	(y,y)
blue	0	0	0	2	2	2	2	2	2	2	2	2
yellow	1	1	0	1	1	0	1	1	0	0	0	0

Round 2:

Recursive Elimination Procedure

Example: When Chris Joins the Party

You	(b,b)	(g,b)	(r,b)	(y,b)	(b,y)	(g,y)	(r,y)	(y,y)
green	3	0	3	3	3	0	3	3
red	2	2	0	2	2	2	0	2
~~*yellow*~~	~~1~~	~~1~~	~~1~~	~~0~~	~~0~~	~~0~~	~~0~~	~~0~~

Barb	(g,b)	(r,b)	(y,b)	(g,y)	(r,y)	(y,y)
blue	0	0	0	3	3	3
green	0	4	4	0	4	4
red	1	0	1	1	0	1
yellow	2	2	0	0	0	0

Chris	(g,b)	(r,b)	(y,b)	(g,g)	(r,g)	(y,g)	(g,r)	(r,r)	(y,r)	(g,y)	(r,y)	(y,y)
blue	0	0	0	2	2	2	2	2	2	2	2	2
yellow	1	1	0	1	1	0	1	1	0	0	0	0

Round 2: For Barbara, *red* is strictly dominated by $(0.4) \cdot$ *blue* + $(0.6) \cdot$ *yellow*.

Recursive Elimination Procedure

Example: When Chris Joins the Party

You	(b,b)	(g,b)	(r,b)	(y,b)	(b,y)	(g,y)	(r,y)	(y,y)
green	3	0	3	3	3	0	3	3
red	2	2	0	2	2	2	0	2
yellow	1	1	1	0	0	0	0	0

Barb	(g,b)	(r,b)	(y,b)	(g,y)	(r,y)	(y,y)
blue	0	0	0	3	3	3
green	0	4	4	0	4	4
red	1	0	1	1	0	1
yellow	2	2	0	0	0	0

Chris	(g,b)	(r,b)	(y,b)	(g,g)	(r,g)	(y,g)	(g,r)	(r,r)	(y,r)	(g,y)	(r,y)	(y,y)
blue	0	0	0	2	2	2	2	2	2	2	2	2
yellow	1	1	0	1	1	0	1	1	0	0	0	0

Round 2: For Barbara, *red* is strictly dominated by $(0.4) \cdot$ *blue* + $(0.6) \cdot$ *yellow*.

Recursive Elimination Procedure

Example: When Chris Joins the Party

You	(b,b)	(g,b)	(r,b)	(y,b)	(b,y)	(g,y)	(r,y)	(y,y)
green	3	0	3	3	3	0	3	3
red	2	2	0	2	2	2	0	2
yellow	1	1	1	0	0	0	0	0

Barb	(g,b)	(r,b)	(y,b)	(g,y)	(r,y)	(y,y)
blue	0	0	0	3	3	3
green	0	4	4	0	4	4
red	1	0	1	1	0	1
yellow	2	2	0	0	0	0

Chris	(g,b)	(r,b)	(y,b)	(g,g)	(r,g)	(y,g)	(g,r)	(r,r)	(y,r)	(g,y)	(r,y)	(y,y)
blue	0	0	0	2	2	2	2	2	2	2	2	2
yellow	1	1	0	1	1	0	1	1	0	0	0	0

Round 3:

Recursive Elimination Procedure

Example: When Chris Joins the Party

You	(b,b)	(g,b)	(r,b)	(y,b)	(b,y)	(g,y)	(r,y)	(y,y)
green	3	0	3	3	3	0	3	3
red	2	2	0	2	2	2	0	2
yellow	1	1	1	0	0	0	0	0

Barb	(g,b)	(r,b)	(y,b)	(g,y)	(r,y)	(y,y)
blue	0	0	0	3	3	3
green	0	4	4	0	4	4
red	1	0	1	1	0	1
yellow	2	2	0	0	0	0

Chris	(g,b)	(r,b)	(y,b)	(g,g)	(r,g)	(y,g)	(g,r)	(r,r)	(y,r)	(g,y)	(r,y)	(y,y)
blue	0	0	0	2	2	2	2	2	2	2	2	2
yellow	1	1	0	1	1	0	1	1	0	0	0	0

Round 3: No choice is strictly dominated.

Procedure terminates.

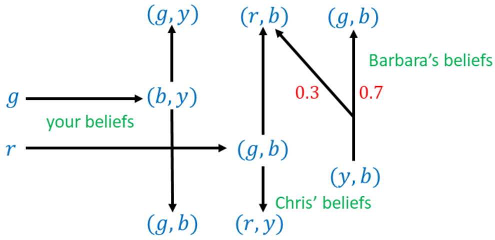
Recursive Elimination Procedure

Example: When Chris Joins the Party

You	(b,b)	(g,b)	(r,b)	(y,b)	(b,y)	(g,y)	(r,y)	(y,y)
green	3	0	3	3	3	0	3	3
red	2	2	0	2	2	2	0	2
yellow	1	1	1	0	0	0	0	0

Barb	(g,b)	(r,b)	(y,b)	(g,y)	(r,y)	(y,y)
blue	0	0	0	3	3	3
green	0	4	4	0	4	4
red	1	0	1	1	0	1
yellow	2	2	0	0	0	0

Chris	(g,b)	(r,b)	(y,b)	(g,g)	(r,g)	(y,g)	(g,r)	(r,r)	(y,r)	(g,y)	(r,y)	(y,y)
blue	0	0	0	2	2	2	2	2	2	2	2	2
yellow	1	1	0	1	1	0	1	1	0	0	0	0



all belief hierarchies express common belief in rationality

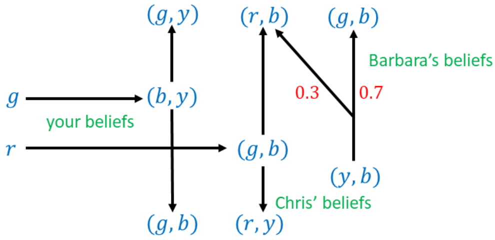
Recursive Elimination Procedure

Example: When Chris Joins the Party

You	(b,b)	(g,b)	(r,b)	(y,b)	(b,y)	(g,y)	(r,y)	(y,y)
green	3	0	3	3	3	0	3	3
red	2	2	0	2	2	2	0	2
yellow	1	1	1	0	0	0	0	0

Barb	(g,b)	(r,b)	(y,b)	(g,y)	(r,y)	(y,y)
blue	0	0	0	3	3	3
green	0	4	4	0	4	4
red	1	0	1	1	0	1
yellow	2	2	0	0	0	0

Chris	(g,b)	(r,b)	(y,b)	(g,g)	(r,g)	(y,g)	(g,r)	(r,r)	(y,r)	(g,y)	(r,y)	(y,y)
blue	0	0	0	2	2	2	2	2	2	2	2	2
yellow	1	1	0	1	1	0	1	1	0	0	0	0



Can be translated into epistemic model where all types express common belief in rationality.

$$b_1(t_1^g) = ((b, t_2^b), (y, t_3^y))$$

and so on.

- In the iterated elimination of strictly dominated choices we must eliminate, at every round, and for every player,

  all states that involve opponents' choices that have been eliminated in the previous round, and subsequently,

  all choices for the player that become strictly dominated.

- Suppose we forget to do some of these eliminations at some of the rounds.

- Will it matter for the eventual output?

- No, as long as we do not forget any elimination forever.

- Order independence.

Recursive Elimination Procedure

Order Independence: Example

You	*blue*	*green*	*red*	*yellow*
blue	0	4	4	4
green	3	0	3	3
red	2	2	0	2
yellow	1	1	1	0

Barbara	*blue*	*green*	*red*	*yellow*
blue	0	2	2	2
green	1	0	1	1
red	4	4	0	4
yellow	3	3	3	0

Round 1: For you, *yellow* is strictly dominated by $(0.5) \cdot$ *blue* + $(0.5) \cdot$ *green*

For Barbara, *green* is strictly dominated by $(0.5) \cdot$ *red* + $(0.5) \cdot$ *yellow*

Recursive Elimination Procedure

Order Independence: Example

You	*blue*	*green*	*red*	*yellow*
blue	0	4	4	4
green	3	0	3	3
red	2	2	0	2
~~*yellow*~~	~~1~~	~~1~~	~~1~~	~~0~~

Barbara	*blue*	*green*	*red*	*yellow*
blue	0	2	2	2
green	1	0	1	1
red	4	4	0	4
yellow	3	3	3	0

Round 1: For you, *yellow* is strictly dominated by $(0.5) \cdot$ *blue* + $(0.5) \cdot$ *green*

For Barbara, *green* is strictly dominated by $(0.5) \cdot$ *red* + $(0.5) \cdot$ *yellow*

Recursive Elimination Procedure

Order Independence: Example

You	*blue*	*green*	*red*	*yellow*
blue	0	4	4	4
green	3	0	3	3
red	2	2	0	2
~~*yellow*~~	~~1~~	~~1~~	~~1~~	~~0~~

Barbara	*blue*	*green*	*red*	*yellow*
blue	0	2	2	2
green	1	0	1	1
red	4	4	0	4
yellow	3	3	3	0

Round 2: Eliminate state *yellow* in Barbara's decision problem.

Recursive Elimination Procedure

Order Independence: Example

You	*blue*	*green*	*red*	*yellow*
blue	0	4	4	4
green	3	0	3	3
red	2	2	0	2
~~*yellow*~~	~~1~~	~~1~~	~~1~~	~~0~~

Barbara	*blue*	*green*	*red*	*yellow*
~~*blue*~~	~~0~~	~~2~~	~~2~~	~~2~~
~~*green*~~	~~1~~	~~0~~	~~1~~	~~1~~
red	4	4	0	4
yellow	3	3	3	0

Round 2: Eliminate choices *blue* and *green* in Barbara's decision problem.

Recursive Elimination Procedure

Order Independence: Example

You	blue	green	red	yellow
blue	0	4	4	4
green	3	0	3	3
red	2	2	0	2
yellow	1	1	1	0

Barbara	blue	green	red	yellow
blue	0	2	2	2
green	1	0	1	1
red	4	4	0	4
yellow	3	3	3	0

Round 3: Eliminate states *blue* and *green* in your decision problem.

Recursive Elimination Procedure

Order Independence: Example

You	*blue*	*green*	*red*	*yellow*
blue	0	4	4	4
green	3	0	3	3
red	2	2	0	2
yellow	1	1	1	0

Barbara	*blue*	*green*	*red*	*yellow*
blue	0	2	2	2
green	1	0	1	1
red	4	4	0	4
yellow	3	3	3	0

Round 3: Eliminate choices *green* and *red* in your decision problem.

Recursive Elimination Procedure

Order Independence: Example

You	*blue*	*green*	*red*	*yellow*
blue	0	4	4	4
green	3	0	3	3
red	2	2	0	2
yellow	1	1	1	0

Barbara	*blue*	*green*	*red*	*yellow*
blue	0	2	2	2
green	1	0	1	1
red	4	4	0	4
yellow	3	3	3	0

Round 4: Eliminate states *green* and *red* in Barbara's decision problem.

Recursive Elimination Procedure

Order Independence: Example

You	*blue*	*green*	*red*	*yellow*
blue	0	4	4	4
green	3	0	3	3
red	2	2	0	2
yellow	1	1	1	0

Barbara	*blue*	*green*	*red*	*yellow*
blue	0	2	2	2
green	1	0	1	1
red	4	4	0	4
yellow	3	3	3	0

Round 4: Eliminate choice *yellow* in Barbara's decision problem.

Same output as with original order of elimination.

Takes one round more.

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