Common Belief in Rationality with Unawareness

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Introduction

- Previous two lectures:
  Uncertainty regarding others' utility functions/conditional preferences.

- This lecture:
  What if players are unaware of (some) choices available to opponents?

- Characteristic feature of unawareness:
  Players cannot reason about events that they are unaware of.
  (Different from and complementary to incomplete information.)

- New tool in this lecture:
  Players hold potentially different views of the game, specifying what choices they believe are available to themselves and opponents.

Introductory Example

Day at the Beach

- *You* and *Barbara* each choose to go to one of four beaches: *Nextdoor, Closeby, Faraway, Distant*.
- Personally, you prefer *Faraway* to *Distant* to *Nextdoor* to *Closeby*. In addition and more importantly, you seek to *avoid* Barbara.
- You know that *Barbara* is aware of *Nextdoor* and *Closeby*, but she may be unaware of *Faraway* and *Distant*.
- You know that Barbara also wants to avoid you and personally prefers *Nextdoor* to *Closeby*.
- Also, **subject to her awareness**, you believe Barbara prefers *Closeby* to *Faraway* to *Distant*.

Introductory Example: Views

v_1^{all} :

You	*Faraway*	*Distant*	*Nextdoor*	*Closeby*
Faraway	0	4	4	4
Distant	3	0	3	3
Nextdoor	2	2	0	2
Closeby	1	1	1	0

v_1^{two} :

You	*Nextdoor*	*Closeby*
Nextdoor	0	2
Closeby	1	0

v_2^{all} :

Barbara	*Faraway*	*Distant*	*Nextdoor*	*Closeby*
Faraway	0	2	2	2
Distant	1	0	1	1
Nextdoor	4	4	0	4
Closeby	3	3	3	0

v_2^{two} :

Barbara	*Nextdoor*	*Closeby*
Nextdoor	0	4
Closeby	3	0

Note: v_1^{two} is needed here. At v_2^{two}, Barbara believes v_1^{two} is your view. Also, she may believe v_1^{two} is your view at v_2^{all}.

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Introductory Example: Rationality

v_1^{all} :

You	*Faraway*	*Distant*	*Nextdoor*	*Closeby*
Faraway	0	4	4	4
Distant	3	0	3	3
Nextdoor	2	2	0	2
Closeby	1	1	1	0

v_1^{two} :

You	*Nextdoor*	*Closeby*
Nextdoor	0	2
Closeby	1	0

- Proceed by eliminating strictly dominated choices at each view.
- At v_1^{all}, $Closeby$ strictly dominated by (e.g.) $1/2 \cdot Faraway + 1/2 \cdot Distant$.
- No choice dominated at v_1^{two}.
- Analogously for v_2^{all}, $Distant$ strictly dominated by (e.g.) $1/2 \cdot Nextdoor + 1/2 \cdot Closeby$ and no choice dominated at v_2^{two}.

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Introductory Example: 1-Fold Strict Dominance

v_1^{all} :

You	*Faraway*	*Distant*	*Nextdoor*	*Closeby*
Faraway	0	4	4	4
Distant	3	0	3	3
Nextdoor	2	2	0	2

v_1^{two} :

You	*Nextdoor*	*Closeby*
Nextdoor	0	2
Closeby	1	0

v_2^{all} :

Barbara	*Faraway*	*Distant*	*Nextdoor*	*Closeby*
Faraway	0	2	2	2
Nextdoor	4	4	0	4
Closeby	3	3	3	0

v_2^{two} :

Barbara	*Nextdoor*	*Closeby*
Nextdoor	0	4
Closeby	3	0

Introductory Example: Belief in Rationality

- Which choices can Barbara and you consider under belief in rationality?
- First consider Barbara's choice *Distant*:
  - At v_2^{all}, *Distant* is never rational for her.
  - At v_2^{two}, she is unaware of *Distant*.

  $\Rightarrow$ You discard *Distant* at v_1^{all} if you believe in Barbara's rationality.
- Now consider your choice *Closeby*:
  - At v_1^{all}, *Closeby* is never rational for you.
  - But at v_1^{two}, *Closeby* is optimal against (e.g.) *Nextdoor*.

  $\Rightarrow$ Barbara does not discard *Closeby* at v_2^{all}!

Introductory Example: Belief in Rationality

v_1^{all} :

You	*Faraway*	*Nextdoor*	*Closeby*
Faraway	0	4	4
Distant	3	3	3
Nextdoor	2	0	2

v_1^{two} :

You	*Nextdoor*	*Closeby*
Nextdoor	0	2
Closeby	1	0

v_2^{all} :

Barbara	*Faraway*	*Distant*	*Nextdoor*	*Closeby*
Faraway	0	2	2	2
Nextdoor	4	4	0	4
Closeby	3	3	3	0

v_2^{two} :

Barbara	*Nextdoor*	*Closeby*
Nextdoor	0	4
Closeby	3	0

Introductory Example: Rationality & Belief in Rat.

v_1^{all} :

You	*Faraway*	*Nextdoor*	*Closeby*
Faraway	0	4	4
Distant	3	3	3
Nextdoor	2	0	2

v_1^{two} :

You	*Nextdoor*	*Closeby*
Nextdoor	0	2
Closeby	1	0

- At v_1^{all}, *Nextdoor* strictly dominated by *Distant*.
- All remaining choices consistent with common belief in rationality.
- **Intuition:**
  - Barbara cannot discard v_1^{two}.
  - Hence, excluding *Nextdoor* and *Closeby* for you at v_1^{all} does not change her decision problems.

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Introductory Example: Rationality & Belief in Rat.

v_1^{all} :

You	*Faraway*	*Nextdoor*	*Closeby*
Faraway	0	4	4
Distant	3	3	3

v_1^{two} :

You	*Nextdoor*	*Closeby*
Nextdoor	0	2
Closeby	1	0

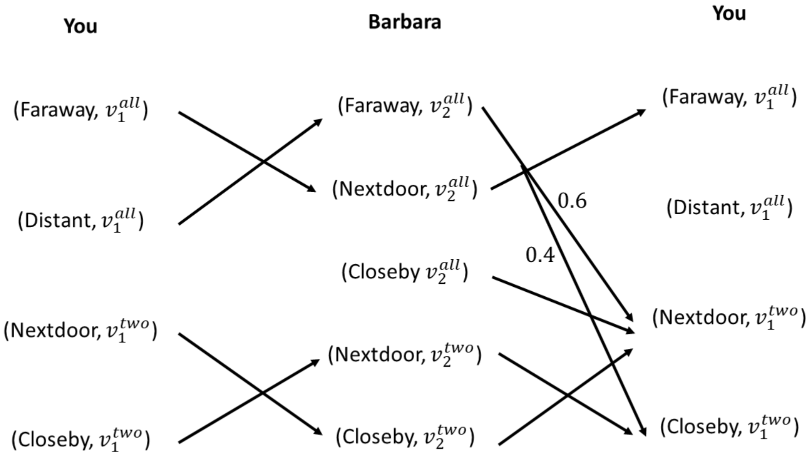
v_2^{all} :

Barbara	*Faraway*	*Distant*	*Nextdoor*	*Closeby*
Faraway	0	2	2	2
Nextdoor	4	4	0	4
Closeby	3	3	3	0

v_2^{two} :

Barbara	*Nextdoor*	*Closeby*
Nextdoor	0	4
Closeby	3	0

Introductory Example: Beliefs Diagram



How does *Nextdoor* and *Closeby* being (iteratively) strictly dominated for you at v_1^{all} affect Barbara's beliefs at v_2^{all}?

Agenda

- Games with Unawareness and Common Belief in Rationality
- Procedural Characterization
- Possibility
- Variants of the Procedure
- Correct and Symmetric Beliefs

Views and Awareness Principle

Definition

A ***view*** for player i specifies a set of choices $C_j(v_i)$ for every player j (including i). If two views $v_i,\ v_k$ satisfy $C_j(v_i) \subseteq C_j(v_k)$ for all players j, then v_i ***is contained in*** v_k.

Important: Players can only reason about choices they are aware of at their view. Hence, any player must believe opponent views are contained in their own view!

Definition

A player with view v satisfies the ***awareness principle*** if they believe that every opponent holds a view contained in v.

Games with Unawareness

Definition

A ***game with unawareness*** specifies

a) finite set I of players,

b) finite collection V_i of views for each player i,

c) utility function $u_i^{v_i} : \times_j C_j(v_i) \to \mathbb{R}$ for every view v_i,

where, for all players i, j,

1) if $v_i \in V_i$, then there is $v_j \in V_j$ such that v_j is contained in v_i,

2) if $(c_i, c_{-i}) \in \times_j C_j(v_i) \cap \times_j C_j(v_i')$, then $u_i^{v_i}(c_i, c_{-i}) = u_i^{v_i'}(c_i, c_{-i})$.

Notes:

(1) ensures reasoning can satisfy awareness principle.

(2) constant utility across views (henceforth write u_i iso $u_i^{v_i}$).

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Beliefs

Under unawareness, players form beliefs about two things:

1) opponents' views,
2) opponents' choices given a view.

(Similar to incomplete info, w. views instead of utilities.)

Formally, **belief hierarchy** for i under unawareness specifies:

- **first-order belief** b_i^1 about opponents' choice-view combinations (c_{-i}, v_{-i}),
- **second-order belief** b_i^2 about opponents' combinations of choices, views, and first-order beliefs $(c_{-i}, v_{-i}, b_{-i}^1)$,

⋮

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Epistemic Models

Definition

Take a game with unawareness Γ. An ***epistemic model*** $M^\Gamma = (T_i, w_i, b_i)_{i \in I}$ specifies

a) finite set of types T_i for every player i,

b) a view $w_i(t_i)$ for every type t_i,

c) a description map $b_i : T_i \to \Delta(C_{-i} \times T_{-i})$,

where, for every players $i \neq j$ and types t_i, t_j,

1) t_i only assigns positive probability to (c_j, t_j) if $c_j \in C_j(w_j(t_j))$,

2) t_i only assigns positive prob. to t_j if $w_j(t_j)$ is contained in $w_i(t_i)$.

Note: (1) + (2) ensure awareness principle holds for every type.

Epistemic Model for Introductory Example

- **Types:** $T_1 = \{t_1^{all,F}, t_1^{all,D}, t_1^{two,N}, t_1^{two,C}\}$, $T_2 = \{t_2^{all,F}, t_2^{all,N}, t_2^{all,C}, t_2^{two,N}, t_2^{two,C}\}$
- **Views for You:** $w_1(t_1) = \begin{cases} v_1^{two}, & \text{if } t_1 \in \{t_1^{two,N}, t_1^{two,C}\}, \\ v_1^{all}, & \text{else.} \end{cases}$
- **Views for Barbara:** $w_2(t_2) = \begin{cases} v_2^{two}, & \text{if } t_2 \in \{t_2^{two,N}, t_2^{two,C}\}, \\ v_2^{all}, & \text{else.} \end{cases}$
- **Beliefs for You:** $b_1(t_1^{all,F}) = (Nextdoor, t_2^{all,N})$, $b_1(t_1^{all,D}) = (Faraway, t_2^{all,F})$,

  $b_1(t_1^{two,N}) = (Closeby, t_2^{two,C})$, $b_1(t_1^{two,C}) = (Nextdoor, t_2^{two,N})$.
- **Beliefs for Barbara:** $b_2(t_2^{all,F}) = 0.6 \cdot (Nextdoor, t_1^{two,N}) + 0.4 \cdot (Closeby, t_1^{two,C})$,

  $b_2(t_2^{all,N}) = (Faraway, t_1^{all,F})$, $b_2(t_2^{all,C}) = (Nextdoor, t_1^{two,N})$,

  $b_2(t_2^{two,N}) = (Closeby, t_1^{two,C})$, $b_2(t_2^{two,C}) = (Nextdoor, t_1^{two,N})$.

Optimal and Rational Choices

Main change under unawareness: optimality is view-dependent.

Definition

Take type t_i with view $w_i(t_i)$, utility $u_i^{w_i(t_i)}$, and first-order belief $b_i^1(t_i)$.
Choice $c_i \in C_i(w_i(t_i))$ is *optimal* for t_i if

$$u_i(c_i, b_i^1(t_i)) = \sum_{c_{-i} \in C_{-i}(w_i(t_i))} b_i^1(t_i)(c_{-i}) u_i(c_i, c_{-i}) \geq u_i(c_i', b_i^1(t_i))$$

for all $c_i' \in C_i(w_i(t_i))$.

- Hence, choice c_i is rational for player i **and** view v_i if there is an epistemic model M such that a type t_i in M **with** $w_i(t_i) = v_i$ can rationally choose c_i.
- Analogous for up to k-fold/common belief in rationality.

(Common) Belief in Rationality

Up to k-fold/common belief in rationality now defined like w/o unawareness:

Definition

Type t_i,

- *believes in the opponents' rationality* if $b_i(t_i)$ only deems possible (c_j, t_j) where c_j is optimal for t_j,
- *expresses up to k-fold belief in rationality* for $k \geq 1$ if $b_i(t_i)$ only deems possible (c_j, t_j) where c_j is optimal for t_j expressing up to $(k-1)$-fold belief in rationality,
- *expresses common belief in rationality* if $b_i(t_i)$ expresses up to k-fold belief in rationality for all $k \geq 1$.

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Strict Dominance and Unawareness

- To find all choices consistent with common belief in rationality under unawareness, we generalize iterated strict dominance.
- Similar to incomplete information and generalized iterated strict dominance, *iterated strict dominance for unawareness* will proceed view by view at each step of the procedure.
- Crucially, we may only discard an opponent j's choice c_j at some view v_i if c_j is strictly dominated at **all** opponent's views v_j contained in v_i such that $c_j \in C_j(v_j)$.

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Strict Dominance for Unawareness

Strict dominance straightforwardly generalizes to unawareness:

Definition

Let Γ be a game with unawareness and take any player i and view $v_i \in V_i$. A choice c_i is *strictly dominated* at v_i if there exists $r \in \Delta(C_i(v_i))$ such that

$$u_i(c_i, c_{-i}) < \sum_{c'_i \in \text{Supp}(r)} r(c'_i) u_i(c'_i, c_{-i})$$

for all $c_{-i} \in C_{-i}(v_i)$.

Using Pearce's Lemma from Chapter 2, we then have:

Theorem

A choice c_i is rational for player i at view v_i iff it is not strictly dominated at v_i.

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Reduced Decision Problems

- As defined above, belief in rationality requires you to only consider opponent (c_j, t_j) combinations s.th. c_j is optimal for t_j.
- But recall that optimality depends on t_j's view $w_j(t_j)$.
- Hence, for any view v_i, we can rule out a choice c_j only if **no** type t_j such that $w_j(t_j)$ is contained in v_i can optimally choose c_j.
- Consequently, eliminating c_j at some view v_i requires that c_j be strictly dominated at **all** views v_j contained in v_i.
- **Note:** Like in introductory example, we allow for player j not being aware of c_j at some of the v_j.

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Iterated Strict Dominance for Unawareness

Definition

Round 1. For every player i and every view v_i, eliminate all choices c_i that are strictly dominated.

Round $k \geq 1$. For every player i and every view v_i, eliminate all opponent choice combinations c_{-i} ($\hat{=}$ states) s.th. some c_j in c_{-i} did not survive round $k-1$ for j at all views v_j contained in v_i. Within the resulting decision problems, for any player i and any view v_i, eliminate all choices c_i that are strictly dominated.

Proceed until no further choices c_i or states c_{-i} can be eliminated at any view v_i of any player i.

Theorem

For any $k \geq 1$, choice c_i is rational for player i at view v_i under up to k-fold (common) belief in rationality iff c_i survives $(k+1)$-fold (iterated) strict dominance for unawareness at v_i.

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Example: “Too much Wine”

- *You* and *Barbara* have drunk too much and (in chronological order) ruined Chris’s *table*, *window*, *roof*, and *door*. Fixing each item will cost \$500.
- Both of you are separately interviewed by Chris to determine responsibility for the damage. Each of you can claim to be *innocent*, or admit to all damages that happened up to destruction of the *table*, *window*, *roof*, and *door*.
- Chris will go with the story admitting to more damages. In addition, if you or Barbara admit to less damages than the other you have to pay an extra \$300, which is used to reward the other player for their honesty.
- *You* are aware of everything that happened. But, due to the wine, you are unsure whether *Barbara* is aware of anything that happened after destruction of the *window*.

"Too much Wine": Views

$v_1^{window} / v_2^{window}$:

You/Barbara	*innocent*	*table*	*window*
innocent	0	-550	-800
table	50	-250	-800
window	-200	-200	-500

v_1^{roof} / v_2^{roof} :

You/Barbara	*innocent*	*table*	*window*	*roof*
innocent	0	-550	-800	-1,050
table	50	-250	-800	-1,050
window	-200	-200	-500	-1,050
roof	-450	-450	-450	-750

v_1^{door} / v_2^{door} :

You/Barbara	*innocent*	*table*	*window*	*roof*	*door*
innocent	0	-550	-800	-1,050	-1,300
table	50	-250	-800	-1,050	-1,300
window	-200	-200	-500	-1,050	-1,300
roof	-450	-450	-450	-750	-1,300
door	-700	-700	-700	-700	-1,000

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"Too much Wine": Rationality

$v_1^{window}/v_2^{window}$:

You/Barbara	*innocent*	*table*	*window*
innocent	0	-550	-800
table	50	-250	-800
window	-200	-200	-500

v_1^{roof}/v_2^{roof} :

You/Barbara	*innocent*	*table*	*window*	*roof*
innocent	0	-550	-800	-1,050
table	50	-250	-800	-1,050
window	-200	-200	-500	-1,050
roof	-450	-450	-450	-750

v_1^{door}/v_2^{door} :

You/Barbara	*innocent*	*table*	*window*	*roof*	*door*
innocent	0	-550	-800	-1,050	-1,300
table	50	-250	-800	-1,050	-1,300
window	-200	-200	-500	-1,050	-1,300
roof	-450	-450	-450	-750	-1,300
door	-700	-700	-700	-700	-1,000

- *innocent* strictly dominated by $0.9 \cdot table + 0.1 \cdot window$ at v_1^{window}.
- *innocent* strictly dominated by $0.95 \cdot table + 0.05 \cdot roof$ at v_1^{roof}.
- *innocent* strictly dominated by $0.95 \cdot table + 0.05 \cdot door$ at v_1^{door}.
- By symmetry, the same is true for Barbara.
- Since *innocent* was eliminated at all views for both players, can eliminate that state from all decision problems.

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"Too much Wine": Belief in Rationality

$v_1^{window}/v_2^{window}$:

You/Barbara	*table*	*window*
table	-250	-800
window	-200	-500

v_1^{roof}/v_2^{roof} :

You/Barbara	*table*	*window*	*roof*
table	-250	-800	-1,050
window	-200	-500	-1,050
roof	-450	-450	-750

v_1^{door}/v_2^{door} :

You/Barbara	*table*	*window*	*roof*	*door*
table	-250	-800	-1,050	-1,300
window	-200	-500	-1,050	-1,300
roof	-450	-450	-750	-1,300
door	-700	-700	-700	-1,000

- *table* strictly dominated by *window* at v_1^{window}.
- *table* strictly dominated by $0.95 \cdot window + 0.05 \cdot roof$ at v_1^{roof}.
- *table* strictly dominated by $0.95 \cdot window + 0.05 \cdot door$ at v_1^{door}.
- By symmetry, the same is true for Barbara.
- Since *table* was eliminated at all views for both players, can eliminate that state from all decision problems.

“Too much Wine”: Up to 2-fold Belief in Rationality

$v_1^{window}/v_2^{window}$:

You/Barbara	*window*
window	-500

v_1^{roof}/v_2^{roof} :

You/Barbara	*window*	*roof*
window	-500	-1,050
roof	-450	-750

v_1^{door}/v_2^{door} :

You/Barbara	*window*	*roof*	*door*
window	-500	-1,050	-1,300
roof	-450	-750	-1,300
door	-700	-700	-1,000

- *window* strictly dominated by *roof* at v_1^{roof}.
- *window* strictly dominated by $0.95 \cdot roof + 0.05 \cdot door$ at v_1^{door}.
- By symmetry, the same is true for Barbara.
- However, *window* will remain a state in each decision problem since it cannot be eliminated at the least expressive views $v_1^{window}/v_2^{window}$!
- Hence, the procedure stops here.

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"Too much Wine": Common Belief in Rationality

$v_1^{window}/v_2^{window}$:

You/Barbara	*window*
window	-500

v_1^{roof}/v_2^{roof} :

You/Barbara	*window*	*roof*
roof	-450	-750

v_1^{door}/v_2^{door} :

You/Barbara	*window*	*roof*	*door*
roof	-450	-750	-1,300
door	-700	-700	-1,000

- At your view v_1^{door}, you can rationally choose $roof$ and $door$ under common belief in rationality.
- Note that rationality of $roof$ under CBR is driven by differential awareness. If you were sure that Barbara is aware of the roof's or the door's destruction, then you could only rationally choose $door$.

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Possibility of Common Belief in Rationality

- An important question is whether games with unawareness are always consistent with common belief in rationality.
- In other words, for any such game Γ, can we find a model M^Γ such that some type t_i for every i expresses common belief in rationality?
- A new variant of this question is whether any **view** in a game is consistent with common belief in rationality.
- In other words, for any Γ, any player i, and any $v_i \in V_i$, can we find a model M^Γ such that some t_i with $w_i(t_i) = v_i$ expresses common belief in rationality?
- We now argue that the answer to both questions is **yes**.

Possibility: Sketch of Proof

- **Step 1:**
  - For every player i and every view v_i, take any $c_{-i} \in C_{-i}(v_i)$.
  - Since $C_i(v_i)$ is finite, there is $c_i^1 \in C_i(v_i)$ s.th. c_i is optimal for $b_i^1 = c_{-i}$.
  - Hence, c_i^1 survives 1-fold str. dominance at v_i.
- **Step $k \geq 1$:**
  - Using Step $k-1$ and awareness principle, for every view v_i and player i, take $c_{-i} \in C_{-i}(v_i)$ s.th. every c_j in c_{-i} is in j's reduced dec. problem after $(k-1)$-fold str. dominance for some v_j contained in v_i.
  - Since $C_i(v_i)$ is finite, there is $c_i^k \in C_i(v_i)$ s.th. c_i is optimal for $b_i^1 = c_{-i}$.
  - Furthermore, since each c_j in c_{-i} is in j's reduced dec. problem after $(k-1)$-fold strict dominance for some v_j contained in v_i, c_{-i} is a state in i's dec. problem at v_i after $(k-1)$-fold str. dominance.
  - Hence, c_i^k survives k-fold strict dominance at v_i.
- Now since there are finitely views and choices, procedure terminates in finitely many steps. Hence, for any view, there must be a choice consistent with common belief in rationality.

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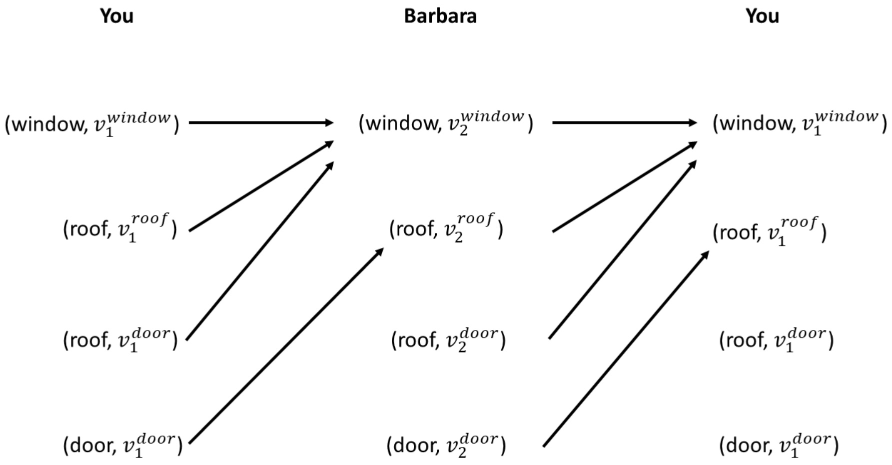
Beliefs Diagrams for Common Belief in Rationality

Similarly, we can find a beliefs diagram supporting any choice c_i that survives iterated strict dominance at some v_i under common belief in rationality:

- Take any view v_i and choice c_i surviving iterated strict dominance at v_i.
- By construction, there is a belief over states in the final reduced problem at v_i that makes c_i optimal. Use this belief for an arrow supporting c_i in the beliefs diagram.
- Again by construction, the arrow only reaches opponent v_j's contained in v_i and c_j's surviving iterated strict dominance at v_j.
- Thus, for each (c_j, v_j) reached by initial arrow, can find belief over states in final reduced decision problem at v_j s.th. c_j is optimal. This yields support arrow for c_j only reaching v_k contained in v_j + c_k surviving iterated str.dom. at v_k.
- Iterating, we arrive at infinite chain of arrows supporting c_i, giving rise to belief hierarchy expressing common belief in rationality.

Example: Beliefs Diagram for “Too much Wine”



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Order Independence

- Similar to standard iterated strict dominance, iterated strict dominance for unawareness is *order-independent*.

- Intuitively, this is true for two reasons:

  1) If a choice is strictly dominated in a decision problem, it is also strictly dominated in any reduced version of that problem.

  2) If a state is not eliminated because strict dominance at some view was overlooked, it can still be eliminated as soon as the necessary strict dominance relationship is detected.

- As a consequence, we can vary the order eliminating choices and states while preserving the **final** output of iterated strict dominance for unawareness.

- **Important:** Correct **intermediate** outputs (k-fold str.dom., $k \geq 1$) only found when eliminating **full-speed** in the **original order**.

Ranked Views

- A computationally convenient order of elimination goes from “smallest” to “largest” views.
- I.e., say that view v **strictly contains** view w if v contains w and $C_j(w) \subsetneq C_j(v)$ for some player j.
- We now rank views in $V = \times_j V_j$ from “smallest” to “largest” as follows:
  - $v \in V$ has rank 1 if no view is strictly contained in v,
  - $v \in V$ has rank 2 if only views of rank 1 are strictly contained in v,
  - $v \in V$ has rank $k \geq 1$ if only views up to rank $k-1$ are strictly contained in v.

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"Too much Wine": Ranked Views

$v_1^{window}/v_2^{window}$:

You/Barbara	*innocent*	*table*	*window*
innocent	0	-550	-800
table	50	-250	-800
window	-200	-200	-500

v_1^{roof}/v_2^{roof} :

You/Barbara	*innocent*	*table*	*window*	*roof*
innocent	0	-550	-800	-1,050
table	50	-250	-800	-1,050
window	-200	-200	-500	-1,050
roof	-450	-450	-450	-750

v_1^{door}/v_2^{door} :

You/Barbara	*innocent*	*table*	*window*	*roof*	*door*
innocent	0	-550	-800	-1,050	-1,300
table	50	-250	-800	-1,050	-1,300
window	-200	-200	-500	-1,050	-1,300
roof	-450	-450	-450	-750	-1,300
door	-700	-700	-700	-700	-1,000

- v_1^{window} is rank 1 for you, v_1^{roof} is rank 2 for you, and v_1^{door} is rank 3 for you.
- The same ranking holds for Barbara.
- More generally, multiple views of different sizes may occupy the same rank (and then they are incomparable).

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Bottom-up Procedure

The following procedure is output-equivalent to the original one:

Definition

Round 1. To all views of rank 1 apply iterated strict dominance for unawareness.

Round $k \geq 1$. For every player i and every view v_i of rank k containing only opponent views of rank $k-1$, eliminate all states involving opponent choices that did not survive step $k-1$ at any view contained in v_i. Now apply iterated strict dominance for unawareness to all views of rank k.

Proceed until all views have been covered.

Theorem

The bottom-up procedure always yields the same final output as iterated strict dominance for unawareness.

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Fixed Beliefs on Views

- An important special case arises from fixing beliefs on views.
- I.e., a player with view v may entertain a fixed probability distribution over opponents' views contained in v, rather than considering any possible distribution over those views.
- Formally within an epistemic model, this becomes a **restriction on the description map**.
- I.e., for all players i and views v_i, let $V_{-i}(v_i) \subseteq V_{-i}$ be opponent views contained in v_i and take a vector $p = (p_i(v_i))_{i \in I, v_i \in V_i}$ s.th. $p_i(v_i) \in \Delta(V_{-i}(v_i))$.
- We can now model **common belief in** p analogous to CBR (i.e., every type believes p, every type believes every type believes p,...). Details in Section 7.6.
- Additional restrictions (e.g., "reverse Bayesianism") or weaker forms (sets of admissible beliefs) can be modeled as well.

Procedures for Fixed Beliefs on Views

- A simple modification to iterated strict dominance delivers choices consistent with CBR **and** common belief in some p.
- Intuitively, the only difference is how decision problems are weighted given beliefs in p.
- As seen in Definition 7.6.4., this leads to a procedure, where for every step $k > 1$, choices survive only if they are optimal for a belief on opponents' choices and views respecting p.
- Otherwise, the procedure stays exactly the same. Also the bottom-up procedure continues to work, subject to the same modification.

Introductory Example with Fixed Beliefs on Views

v_1^{all} :

You	*Faraway*	*Distant*	*Nextdoor*	*Closeby*
Faraway	0	4	4	4
Distant	3	0	3	3
Nextdoor	2	2	0	2
Closeby	1	1	1	0

v_1^{two} :

You	*Nextdoor*	*Closeby*
Nextdoor	0	2
Closeby	1	0

v_2^{all} :

Barbara	*Faraway*	*Distant*	*Nextdoor*	*Closeby*
Faraway	0	2	2	2
Distant	1	0	1	1
Nextdoor	4	4	0	4
Closeby	3	3	3	0

v_2^{two} :

Barbara	*Nextdoor*	*Closeby*
Nextdoor	0	4
Closeby	3	0

- Suppose you assign $p_1(v_1^{all}) = 0.8 \cdot v_2^{all} + 0.2 \cdot v_2^{two}$ and that Barbara assigns the same weighting to v_1^{all}, v_1^{two} at v_2^{all}.
- Clearly, this does not matter for rationality. I.e., *Closeby* at v_1^{all} and *Distant* at v_2^{all} remain the only strictly dominated choices.

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Introductory Example: Belief in Rationality

- Which choices can Barbara and you make under belief in rationality, given the belief restriction?
- *Nextdoor* will still be eliminated for you, given that *Distant* was eliminated at **all** of Barbara's views.
- Now consider Barbara's choice *Faraway* at v_2^{all}:
  - At v_2^{all}, Barbara must believe you do not choose *Closeby*.
  - Hence, at v_2^{all}, Barbara believes you choose *Closeby* with probability at most 0.2 (if your view is v_1^{two}).
  - But then, Barbara expects at least $0.8 * 3 = 2.4$ when choosing *Closeby* and at most 2 when choosing *Faraway*.
  - So *Faraway* is eliminated for Barbara!

Fixed Beliefs: Up to 2-fold Belief in Rationality

v_1^{all} :

You	*Nextdoor*	*Closeby*
Faraway	4	4
Distant	3	3

v_1^{two} :

You	*Nextdoor*	*Closeby*
Nextdoor	0	2
Closeby	1	0

v_2^{all} :

Barbara	*Faraway*	*Distant*	*Nextdoor*	*Closeby*
Nextdoor	4	4	0	4
Closeby	3	3	3	0

v_2^{two} :

Barbara	*Nextdoor*	*Closeby*
Nextdoor	0	4
Closeby	3	0

- *Distant* strictly dominated for you at v_1^{all}.
- Moreover, at v_2^{all}, Barbara must assign probability 0.8 to $\{Faraway, Distant\}$.
- But then, *Nextdoor* yields at least 3.2 and *Closeby* at most 3. So *Closeby* is eliminated.

Fixed Beliefs: Common Belief in Rationality

v_1^{all} :

You	*Nextdoor*	*Closeby*
Faraway	4	4

v_1^{two} :

You	*Nextdoor*	*Closeby*
Nextdoor	0	2
Closeby	1	0

v_2^{all} :

Barbara	*Faraway*	*Distant*	*Nextdoor*	*Closeby*
Nextdoor	4	4	0	4

v_2^{two} :

Barbara	*Nextdoor*	*Closeby*
Nextdoor	0	4
Closeby	3	0

So *Faraway* is your unique choice under common belief in rationality and in the belief restriction!

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Agenda

- Games with Unawareness and Common Belief in Rationality
- Procedural Characterization
- Possibility
- Variants of the Procedure
- Correct and Symmetric Beliefs

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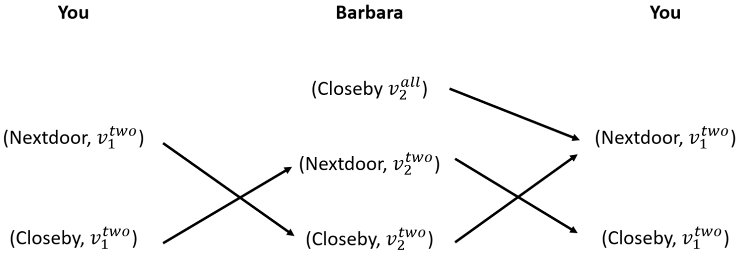
Correct and Symmetric Beliefs

- Similar to (in)complete information, one may wonder about unawareness analogous of Nash- and correlated-equilibrium.
- It turns out that both concepts are trivially equivalent to their complete information counterparts here.
- To see this, suppose type t_i with view $w_i(t_i)$ has symmetric beliefs over choices and views. Then, for any (c_j, v_j) deemed possible by t_i, t_i must believe that some $(c_i, w_i(t_i)$ is deemed possible by player j at v_j.
- But then, v_j must contain $w_i(t_i)$. Since we started from an arbitrary view and arbitrary players, this means that **all** views must contain each other under symmetric beliefs.

$\Rightarrow$ Back to standard games!

Outlook: Weaker Equilibrium Notions

- **Note:** The previous does not preclude weaker forms of equilibria with differential awareness.
- E.g., take "Day at the Beach" with v_1^{two} and v_2^{all}:



- Here, you express CBR and you are correct about Barbara's choice (and vice versa for Barbara). But you may wrongly believe in v_2^{two}.
- Crucially, this happens because Barbara has no incentive to take any of her choices that you are unaware of.