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# Commitment in alternating offers bargaining

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## HIGHLIGHTS

- We extend the alternating-offer bargaining model.
- At the start of each bargaining round, each party may commit to a share of the pie.
- When commitment costs are small but increasing, there is a second mover advantage.
- This reverses the sharing of Rubinstein (1982).

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## ABSTRACT

We extend the Ståhl–Rubinstein alternating-offer bargaining procedure to allow players to simultaneously and visibly commit to some share of the pie prior to, and for the duration of, each bargaining round. If commitment costs are small but increasing in the committed share, then the unique subgame perfect equilibrium outcome exhibits a second mover advantage. In particular, as the horizon approaches infinity, and commitment costs approach zero, the unique bargaining outcome corresponds to the reversed Rubinstein outcome $(\delta/(1+\delta), 1/(1+\delta))$, where $\delta$ is the common discount factor.



## 1. Introduction

"*…it has not been uncommon for union officials to stir up excitement and determination on the part of the membership during or prior to a wage negotiation. If the union is going to insist on* \$2 *and expects the management to counter with* \$1.60, *an effort is made to persuade the membership not only that the management could pay* \$2 *but even perhaps that the negotiators themselves are incompetent if they fail to obtain close to* \$2. *The purpose …is to make clear to the management that the negotiators could not accept less than* \$2 *even if they wished to because they no longer control the members or because they would lose their own positions if they tried*" .

In this quotation from his classic book, Schelling (1960) vividly illustrates that strategic commitment is often an essential feature of bargaining tactics. Parties of negotiations often have access to actions that commit them to some strategically chosen bargaining position. The present paper builds on formal game theoretic analysis inspired by Schelling's work (Crawford, 1982; Muthoo, 1992, 1996; Li, 2011; Ellingsen and Miettinen, 2008, 2014). We analyze the effect of commitment strategies in a dynamic complete information bargaining framework. We limit attention to the finite horizon alternating offer game (Ståhl, 1972; Rubinstein, 1982) although we do study the infinite horizon limit. We model parties who can, simultaneously prior to each offer-response stage, commit not to agree on any share smaller than specified in the commitment. Strategic commitment is assumed to incur small costs. These costs are increasing in the share to which the party commits. This reflects the idea that more resources must be invested to build a credible commitment when the opportunity cost of turning down a deal is larger. After each round of bargaining, any prior commitments are relaxed and players may again choose a commitment to any share they wish. Such short-term commitments are conceivable in delegated bargaining, for instance, where each principal sets limits to acceptable deals for the delegates taking part in the upcoming negotiation round; after a failure to agree the parties leave the negotiation table and receive new guidelines from the principal.[1]

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[1] Maskin and Tirole (1988) study short-run commitments in an alternating-move Cournot-duopoly setting; Vickers (1985) analyzes the role of delegation and vertical structures on commitment.

Our work builds heavily upon the work of Rubinstein (1982) and Ellingsen and Miettinen (2008). The main insight of Rubinstein's pioneering work on bargaining is that, under complete information, equilibrium strategies are determined by the relative impatience of the bargaining parties. In equilibrium, the proposer makes an offer so that the responder is indifferent between accepting the offer and rejecting it, given the cost of waiting; and the responder accepts the offer. Thus there is an efficient immediate agreement with a first-mover advantage. Ellingsen and Miettinen (2008) illustrated how mutual attempts of aggressive incompatible commitment may be unavoidable in bilateral bargaining, if commitments are irreversible and can only be attempted prior to the negotiations, if the costs of committing are relatively small, and if the attempts to commit are not certain to succeed.

In the present context of reversible commitments we show that, in line with Rubinstein (1982) and contrary to Ellingsen and Miettinen (2008), the deal is always stroke immediately. However, contrary to Rubinstein's outcome, there is a second-mover advantage rather than a first-mover advantage even if both parties commit simultaneously at the beginning of every round and there are no exogenous asymmetries in the commitment technology. The intuition for the result is the following: commitments are short-lasting and there is no uncertainty about who has the initiative (pre-determined alternating offers structure). Thus, the first one to propose does not need to commit before her proposal; whatever share the second-mover commits to, it is best for the first-mover to avoid any costs by refraining from committing. In equilibrium she proposes the second-mover the share this latter commits to and takes the residual share herself, provided that the residual makes her better off than waiting for the follow-up round. Knowing this, the second-mover will commit up to the share that makes the first-mover indifferent between having the residual share and waiting. Thus, the presence of symmetric commitment opportunities reverses the bargaining power of the parties. In the limit, where the cost of commitment approaches zero and the number of rounds approaches infinity, the outcome approaches the reversed Rubinstein (1982) outcome, $(\delta/(1+\delta), 1/(1+\delta))$, where $\delta$ is the common discount factor. It is crucial in the analysis that commitments are short-lasting and thus current commitments do not impact the expected future sharing, only the sunk commitment costs. If commitments were expected to bind also in the future rounds, as in Ellingsen and Miettinen (2008, 2014), inefficiencies would result.

Our analysis contributes to the agenda, initiated by Schelling (1956), of carefully analyzing and understanding commitment institutions and mechanisms and their implications on the bargaining outcomes. Among the related works, Crawford (1982) and Muthoo (1992, 1996) have studied the effects of revocable commitments; in our model in contrast, commitments automatically vanish but parties can make a costly recommitment to any share they like after each round. Ellingsen and Miettinen (2008, 2014) analyze costly and long-lasting precommitment to offers and thus they cannot be freely adjusted after each round. Also, unlike in Ellingsen and Miettinen (2008, 2014), players do not commit directly to proposals in our model, but rather to veto any deal where their share is smaller than their commitment. In this respect the model resembles those of Muthoo (1992, 1996) and the endogenous commitment models analyzed in Fershtman and Seidmann (1993), Li (2007), and Miettinen (2010), in which yet, the smallest acceptable shares are determined by the bargaining history in some exogenously determined way rather than freely chosen by players. Related evolutionary analyses of bargaining by Ellingsen (1997), Huck and Oechssler (1999) and Huck et al. (2005) show that exogenous commitment to reject small offers improves evolutionary fitness.

Schelling also mentions reputation as an important means of pre-commitment. Myerson (1991), Kambe (1999), Abreu and Gul (2000) and Wolitzky (2012) analyze reputation contexts where one party has incomplete information about the opponent's stubbornness not to accept anything less than an exogenously given share of the pie. The opponent can then use commitment tactics that exploit this incomplete information and strategically mimic stubbornness in order to force concessions from the other party. This induces delay and influences the final sharing.

Outside options bear a close relation to the current complete information alternating offer bargaining model. Compte and Jehiel (2002) show that exogenous outside options may altogether eliminate the strategic effects of reputation for stubbornness. It has also been shown that, when a party, by opting out, gets a payoff that is inferior to the equilibrium payoff he would obtain in the game without outside options, then these latter have no effect on the equilibrium outcomes (Binmore et al., 1989). In our setting deliberately chosen commitment strategies influence bargaining outcomes exactly because they are chosen so as to force concessions superior to those in the Rubinstein outcome. Ponsati and Sákovics (1998) have shown that if each bargaining party can impose a commonly known inefficient outside option outcome when a deal is rejected, then there are multiple equilibria in the alternating offers game and inefficiencies are possible. Their contribution can be seen as part of the literature studying the robustness of the Rubinstein (unique) outcome to plausible variants of the initial alternating offer game (see Avery and Zemsky, 1994, for a non-exhaustive but inspiring synthesis).

Outside the realm of bargaining, the work of Schelling has inspired a literature analyzing commitment in games in general (Bagwell, 1995; Romano and Yildirim, 2005; Bade et al., 2009; Renou, 2009). In industrial organization, excess capacity provides commitment power in a manner somewhat similar to our bargaining model (Spence, 1977, 1979). Excess investments have a more irreversible nature than the short term bargaining commitments in the present model. Dixit's (1980) extension of the Spence–Dixit excess capacity model shows that an incumbent firm, who nevertheless is presumed to play the role of the follower, can use the commitment, provided by an excess capacity investment, in seizing limited initiative back from the entrant. Ellingsen (1995) shows in a Cournot duopoly setting that, if one of the firms alone can choose to pile up investment later, that firm will endogenously end up in the Stackelberg follower position, whereas the firm who can only invest at present will become the leader.[2] Hamilton and Slutsky (1990) show how Stackelberg outcomes arise by endogenous timing in duopoly games. Whether commitments are short- or long-lasting plays a key role also in infinite horizon settings (Fudenberg and Tirole, 1983; Maskin and Tirole, 1987, 1988). While in capacity commitment models, apart from fixed costs, flow payoffs are typically continuous in capacities, bargaining models exhibit payoff-discontinuities in commitments since just compatible commitments result in an agreement while even small incompatibilities result in delay or disagreement.[3]

The paper is organized as follows. In Section 2 we set up the model and the bargaining procedure. In Section 3 we analyze the model with one round of bargaining. We will use it as a benchmark for our analysis of more than one round in Section 4. We also investigate the limit behavior of this outcome, when the commitment costs go to zero and the number of rounds goes to infinity. We conclude in Section 5.

[2] These are the only strategies surviving iterated elimination of weakly dominated strategies.

[3] Notice that marginal profits with respect to production-stage outputs are discontinuous in Dixit (1980) and so are the slopes of the output reaction curves, but the profits exhibit no discontinuities in *capacity commitments* apart from the potential fixed cost of building capacity. In bargaining models payoffs are discontinuous in *commitments* when the commitments are just-compatible.

## 2. The bargaining procedure

There are two players, 1 and 2, who must reach an agreement about the division of one unit of some good. Let $X := [0, 1]$. Hence, the set of possible divisions is given by

$$D := \{(x_1, x_2) : x_1, x_2 \in X \text{ and } x_1 + x_2 \leq 1\}.$$

Players 1 and 2 use the following bargaining procedure, which can last for at most $N$ rounds.

*Round* 1: At the beginning, both players simultaneously choose *commitment levels* $c_1, c_2 \in X$. The commitment levels become known to both players, and player 1 proposes a division $(x_1, x_2) \in D$ with $x_1 \geq c_1$. Subsequently, player 2 decides whether to accept or reject the proposal under the condition that he can only accept offers with $x_2 \geq c_2$. If he accepts, $(x_1, x_2)$ is the final outcome. If he rejects, the game moves to round 2.

*Round* 2: At the beginning, both players simultaneously choose new commitment levels $c_1, c_2 \in X$. Afterwards, player 2 proposes a division $(x_1, x_2) \in D$ with $x_2 \geq c_2$. Subsequently, player 1 decides whether to accept or reject $(x_1, x_2)$, under the condition that he can only accept offers with $x_1 \geq c_1$. If he accepts, $(x_1, x_2)$ is the final outcome. If he rejects, the game moves to round 3.

*Round* 3: This is a repetition of round 1. And so on.

This bargaining procedure goes on until an agreement is reached, or the process enters round $N + 1$. In round $N + 1$, a given division $(y_1, y_2) \in D$ is realized.

We assume that both players incur a cost for commitment, and that this cost is increasing in the amount to which the player commits. The reason for the latter is that the higher the amount to which the player commits, the more difficult it will be to stick to this commitment. More precisely, if player $i$ commits to an amount $c_i$, this will cost him $\lambda c_i$, where $\lambda$ is some small positive number. For convenience, we assume that $\lambda$ is the same for both players. We finally assume that both players discount future payoffs by a common discount factor $\delta$.

So, in view of all the above, the players' utilities are as follows: If the players reach an agreement on division $(x_1, x_2)$ in round $n$, then the utility for player $i$ is

$$\delta^{n-1} x_i - \lambda \left(c_i^1 + \delta c_i^2 + \cdots + \delta^{n-1} c_i^n\right),$$

where $c_i^k$ is the commitment level chosen at round $k$. If the game reaches round $N + 1$, his utility would be

$$\delta^N y_i - \lambda \left(c_i^1 + \delta c_i^2 + \cdots + \delta^{N-1} c_i^N\right).$$

Throughout the paper, we will assume that $\lambda < 1 - \delta$. That is, the marginal cost of commitment is sufficiently small. We need this assumption in order to establish our main results in Theorems 1 and 2.

Within our bargaining procedure above, the interpretation of the commitment levels is thus that the proposer commits to never offer less than his commitment level for himself, whereas the responder commits to reject any offer that would give him less than his commitment level. With this interpretation in mind, it makes intuitive sense that the cost of commitment is assumed to be increasing in the commitment level. A higher commitment level, namely, more heavily restricts the subsequent choice set of the player, and for higher commitment levels, makes it more tempting for this player to break his commitment. The higher cost of commitment for larger shares should in this way reflect the larger opportunity cost.

## 3. The case of one round

We now analyze the bargaining procedure by using the concept of *subgame perfect equilibrium* (Selten, 1965), where we restrict to pure strategies only. The following tie-breaking rule will be adopted: if a player is indifferent between accepting and rejecting an offer, he is assumed to accept. We start with the easiest case, namely when there is only one round of bargaining. For this case, we already encounter a surprising result: In the unique subgame perfect equilibrium outcome, the proposer faces a first-mover *disadvantage*,rather than a first-mover advantage. Actually, we can say a little more, namely the proposer gets exactly what he would obtain as a responder in the procedure without commitment. So, introducing the possibility to commit reverses the outcome completely.

**Theorem 1** (*Case of One Round*). *Consider the procedure with only one round of bargaining. Then, there is a unique subgame perfect equilibrium outcome, where player* 1 *chooses commitment level* 0*, player* 2 *chooses commitment level* $1 - \delta y_1$*, player* 1 *proposes* $(\delta y_1, 1 - \delta y_1)$ *and player* 2 *accepts.*

Before presenting the proof, let us briefly discuss the result above, and relate it to the traditional setting without commitment. Remember that $(y_1, y_2)$ is the outcome if the proposal is rejected. So, player 1, the proposer, gets the minimal amount he would still accept, whereas player 2, the responder, gets all the surplus. Notice that in the classical bargaining procedure without commitment, this would be exactly the outcome when player 2 would be the proposer and player 1 the responder. In other words, the introduction of commitment "reverses" the traditional outcome between the proposer and the responder — the proposer in the model with commitment gets what he would have obtained as a responder in the setting without commitment, and vice versa. In particular, the proposer in the model with commitment now faces a first-mover disadvantage instead of a first-mover advantage.

The intuition for this "outcome reversal" is that in a setting with commitment, the best that the proposer can do when proposing is to offer player 2 the minimum amount that this latter would still accept — taking into account the commitment level chosen by player 2, and the division $(y_1, y_2)$ in case of a rejection. Suppose this leads to a division $(x_1, x_2)$ that player 2 would accept. But if player 1 can offer this division after having chosen a strictly positive commitment level $c_1 > 0$, he could also have offered the same division $(x_1, x_2)$ after having chosen a commitment level of 0, save the commitment cost, and still be sure that player 2 will accept the offer. Hence, the best that player 1 can do is to choose a commitment level of 0. But then, if player 2 knows this, he will choose his commitment level equal to the maximum amount that player 1 is still willing to offer to player 2 — which is $1 - \delta y_1$. Indeed, player 1 can guarantee an outcome of $\delta y_1$ by having his proposal rejected, and hence the minimum amount he wishes to keep for himself is $\delta y_1$. Hence, in the setting with commitment, the responder can extract all the surplus from the proposer by choosing his commitment level equal to $1 - \delta y_1$, knowing that the proposer will choose a commitment level of 0.

Observe also that in the setting with commitment, the outcome is not efficient as player 2 faces a strictly positive commitment cost $\lambda(1 - \delta y_1)$. On the other hand, no part of the good gets wasted as eventually the full good is divided between players 1 and 2.

**Proof.** For every pair $(c_1, c_2)$ of commitment levels, the subgame that starts after $(c_1, c_2)$ is a game with perfect information. After every $(c_1, c_2)$, the unique subgame perfect equilibrium outcome is as follows:

1. If $c_1 + c_2 > 1$, or $c_1 > 1 - \delta y_2$, then player 2 will reject any proposal by player 1. Hence, the outcome will be $(y_1, y_2)$, with utility $\delta y_1 - \lambda c_1$ for player 1, and utility $\delta y_2 - \lambda c_2$ for player 2.

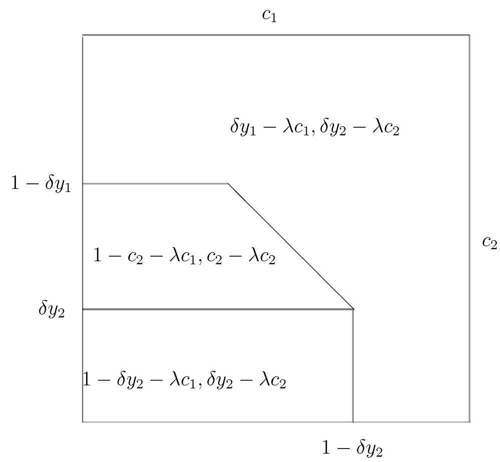


**Fig. 1.** The case of one round: Subgame perfect equilibrium utilities after every pair $(c_1, c_2)$.

2. If $c_2 > 1 - \delta y_1$, then player 1 does not want to make any offer that player 2 would accept. Hence, the outcome will be $(y_1, y_2)$, with utility $\delta y_1 - \lambda c_1$ for player 1, and utility $\delta y_2 - \lambda c_2$ for player 2.
3. Suppose that $c_1 + c_2 \leq 1$ and $\delta y_2 < c_2 \leq 1 - \delta y_1$. Then, the best that player 1 can do is to offer player 2 precisely $c_2$, which player 2 would accept. So, the outcome would be $(1 - c_2, c_2)$, with utility $1 - c_2 - \lambda c_1$ for player 1, and utility $c_2 - \lambda c_2$ for player 2.
4. Suppose that $c_1 \leq 1 - \delta y_2$ and $c_2 \leq \delta y_2$. Then, the best that player 1 can do is to offer player 2 exactly $\delta y_2$, which player 2 would accept (by the tie-breaking rule). So, the outcome would be $(1 - \delta y_2, \delta y_2)$, with utility $1 - \delta y_2 - \lambda c_1$ for player 1, and utility $\delta y_2 - \lambda c_2$ for player 2.

It can easily be seen that this covers all possible cases. In Fig. 1 we have depicted the unique subgame perfect equilibrium utilities for both players after every possible pair $(c_1, c_2)$.

Let us now turn to the beginning of the game, where players 1 and 2 must simultaneously choose the commitment levels $c_1$ and $c_2$. From player 1's utilities in Fig. 1 it may first be verified that, for every $c_2$, player 1's utility is strictly decreasing in his commitment level $c_1$.

To see this, consider first the case where $c_2 \leq \delta y_2$. Then, player 1's utility is given by $v_1(c_1) := 1 - \delta y_2 - \lambda c_1$ when $c_1 \leq 1 - \delta y_2$, and is given by $\hat{v}_1(c_1) := \delta y_1 - \lambda c_1$ when $c_1 > 1 - \delta y_2$. So, clearly, player 1's utility is strictly decreasing in $c_1$ for $c_1 \leq 1 - \delta y_2$ and for $c_1 > 1 - \delta y_2$. To show that it is strictly decreasing in $c_1$ overall, we must verify that $v_1(1 - \delta y_2) \geq \hat{v}_1(1 - \delta y_2)$. Since $v_1(1 - \delta y_2) = 1 - \delta y_2 - \lambda(1 - \delta y_2)$ and $\hat{v}_1(1 - \delta y_2) = \delta y_1 - \lambda(1 - \delta y_2)$ it suffices to show that $1 - \delta y_2 \geq \delta y_1$. But this is true, since $\delta y_1 + \delta y_2 \leq y_1 + y_2 \leq 1$. So, we may conclude that player 1's utility is strictly decreasing in $c_1$ when $c_2 \leq \delta y_2$.

Consider next the case where $c_2 > \delta y_2$ and $c_2 \leq 1 - \delta y_1$. Then, player 1's utility is equal to $v_1(c_1) := 1 - c_2 - \lambda c_1$ when $c_1 \leq 1 - c_2$, and player 1's utility is equal to $\hat{v}_1(c_1) := \delta y_1 - \lambda c_1$ when $c_1 > 1 - c_2$. So, clearly, player 1's utility is strictly decreasing in $c_1$ for $c_1 \leq 1 - c_2$ and for $c_1 > 1 - c_2$. To show that it is strictly decreasing in $c_1$ overall, we must show that $v_1(1 - c_2) \geq \hat{v}_1(1 - c_2)$. By definition, $v_1(1 - c_2) = 1 - c_2 - \lambda(1 - c_2)$ and $\hat{v}_1(1 - c_2) = \delta y_1 - \lambda(1 - c_2)$. So, it suffices to show that $1 - c_2 \geq \delta y_1$. This, however, is true because $c_2 \leq 1 - \delta y_1$ by assumption. Hence, we conclude that player 1's utility is strictly decreasing in $c_1$ when $c_2 > \delta y_2$ and $c_2 \leq 1 - \delta y_1$.

Consider finally the case where $c_2 > 1 - \delta y_1$. Then, player 1's utility is given by $\delta y_1 - \lambda c_1$, which is clearly strictly decreasing in $c_1$.

So, we conclude that, for every $c_2$, player 1's utility is strictly decreasing in his commitment level $c_1$.

This means, however, that $c_1 = 0$ is the unique optimal choice for player 1 at the beginning of the game. But then, it may be verified that player 2's best choice is $c_2 = 1 - \delta y_1$.

To see this, note that player 2's utility when $c_1 = 0$ is given by $v_2(c_2) := \delta y_2 - \lambda c_2$ when $c_2 \leq \delta y_2$, is given by $\hat{v}_2(c_2) := c_2 - \lambda c_2$ when $c_2 > \delta y_2$ and $c_2 \leq 1 - \delta y_1$, and is given by $\tilde{v}_2(c_2) := \delta y_2 - \lambda c_2$ when $c_2 > 1 - \delta y_1$. So, it is clear that player 2's utility is strictly decreasing in $c_2$ for $c_2 \leq \delta y_2$, is strictly increasing in $c_2$ when $c_2 > \delta y_2$ and $c_2 \leq 1 - \delta y_1$, and is strictly decreasing in $c_2$ when $c_2 > 1 - \delta y_1$. Hence, the only candidates for an optimal $c_2$ are $c_2 = 0$ and $c_2 = 1 - \delta y_1$. We will show that $\hat{v}_2(1 - \delta y_1) > v_2(0)$. Note that $\hat{v}_2(1 - \delta y_1) = (1 - \lambda)(1 - \delta y_1)$ and that $v_2(0) = \delta y_2$. Hence, we must show that $(1 - \lambda)(1 - \delta y_1) > \delta y_2$.

We have that

$$\begin{aligned}
\delta y_2 &< \delta y_2 + 1 - \delta - \lambda \\
&\leq \delta(1 - y_1) + 1 - \delta - \lambda \\
&= 1 - \delta y_1 - \lambda \\
&\leq 1 - \delta y_1 - \lambda(1 - \delta y_1) \\
&= (1 - \lambda)(1 - \delta y_1),
\end{aligned}$$

as we had to show. Here, the second inequality follows from the assumption that $\lambda < 1 - \delta$, whereas the third inequality follows from the fact that $y_1 + y_2 \leq 1$, and hence $y_2 \leq 1 - y_1$.

So, we may conclude that, indeed, $\hat{v}_2(1 - \delta y_1) > v_2(0)$. Hence, the best commitment level for player 2, when $c_1 = 0$, is $c_2 = 1 - \delta y_1$.

As we have seen above, the best that player 1 can do in this case is to propose $(\delta y_1, 1 - \delta y_1)$, which player 2 would accept. So, in the unique subgame perfect equilibrium outcome, player 1 chooses commitment level $c_1 = 0$, player 2 chooses $c_2 = 1 - \delta y_1$, player 1 proposes $(\delta y_1, 1 - \delta y_1)$ and player 2 accepts. This completes the proof. ■

Theorem 1 illustrates two points. First, by setting $y_1 = y_2 = 0$, one can see that in a single round ultimatum bargaining game, the second mover will reap the entire pie. Second, by setting $y_1 = \delta/(1 + \delta)$ and $y_2 = 1/(1 + \delta)$, we would effectively add a simultaneous move commitment stage to the alternating-offer protocol such that precommitments are valid only in the first round of bargaining. Our result shows that this would in fact put the recipient of the first offer in an even more advantageous position than where the proposer in the game without commitments is: the recipient of the first proposal commits to $1 - \delta^2/(1 + \delta) = \frac{1+\delta-\delta^2}{1+\delta}$ and leaves only $\delta^2/(1 + \delta)$ to the first mover.

The model is inspired by the model of Ellingsen and Miettinen (2008) and thus the differences are worth discussing. While in the present model we consider short-run commitments that last only for one negotiation round, the commitments are longer-lasting in Ellingsen and Miettinen (2008): when player $j$ commits to a larger share than $1 - y_i$ in the long-run commitment model, it is assumed that player $j$ cannot agree to $1 - y_i$ even at time 2 if the negotiations will have broken down. This is contrary to the present model where failing to agree now results in the payoffs $y_1$ and $y_2$ at time 2. Thus with long-run commitments, if player $i$ does not give in and does not agree on conceding $c_j > 1 - y_i$ to player $j$, both players' payoffs will be zero. The long-term commitment thus gives player $j$ more strategic leverage than a short term commitment. This explains why in Ellingsen and Miettinen the unique iteratively undominated commitment strategy is to commit to accept no less than the entire pie. Notice that in the present model, this is precisely the optimal commitment of player 2 when $y_1 = 0$, i.e. when player 1's continuation payoff is zero (as is assumed in Ellingsen and Miettinen, 2008). Another point worth noting is that if there were any chance that player 2 might be the proposer in the current negotiation round, then even player 1 would have an incentive to commit to force concessions in the event that player 2 is the proposer.

Binmore et al. (1989) discuss the differences between impasse payoffs (the payoffs $y_1$ and $y_2$ at time 2 in case of a breakdown) and outside options that are implied when a player unilaterally chooses to opt out during the course of negotiations. Clearly, in the one-round case we analyzed in this section, it does not matter whether payoffs $y_1$ and $y_2$ are considered as outside option payoffs or breakdown payoffs as long as the outside options are available whether or not commitments have been made. When choosing their commitments, players must ensure that one's commitment leaves the opponent weakly better off than the opponent's outside option. If our model had both outside options and breakdown payoffs, then the responder's optimal commitment would equal the maximum of the proposer's impasse and outside option payoffs.

## 4. The case of more rounds

We now turn to the case of more than one round. Also in this case, the subgame perfect equilibrium leads to a unique outcome, where the proposer at round 1 faces a first-mover *disadvantage,*rather than a first-mover advantage. Actually, when the commitment cost $\lambda$ tends to zero, then the first proposer gets exactly what he would obtain as the first responder in the procedure without commitment, and *vice versa*. So, again, introducing the possibility to commit completely reverses the outcome as $\lambda$ tends to zero.

**Theorem 2** (*Case of More Than One Round*). *Suppose that the bargaining procedure consists of N potential rounds. We define the proposals* $(x_1^{N,k}, x_2^{N,k})$ *with* $k \in \{1, \ldots, N\}$ *as follows:*

*If N is odd, then* $x_1^{N,N} := \delta y_1$ *and* $x_2^{N,N} := 1 - \delta y_1$.

*If N is even, then* $x_1^{N,N} := 1 - \delta y_2$ *and* $x_2^{N,N} := \delta y_2$.

*For every* $k \in \{1, \ldots, N-1\}$, *let the proposal* $(x_1^{N,k}, x_2^{N,k})$ *be given by the following recursive formula:*

*If k is odd, then* $x_1^{N,k} := \delta(1-\lambda)x_1^{N,k+1}$ *and*
$x_2^{N,k} := 1 - \delta(1-\lambda)x_1^{N,k+1}$.

*If k is even, then* $x_1^{N,k} := 1 - \delta(1-\lambda)x_2^{N,k+1}$ *and*
$x_2^{N,k} := \delta(1-\lambda)x_2^{N,k+1}$.

*Then, there is a unique subgame perfect equilibrium, where at every odd round k player 1 commits to* $c_1 = 0$, *player 2 commits to* $c_2 = x_2^{N,k}$, *player 1 proposes* $(x_1^{N,k}, x_2^{N,k})$ *and player 2 accepts, and where at every even round k player 2 commits to* $c_2 = 0$, *player 1 commits to* $c_1 = x_1^{N,k}$, *player 2 proposes* $(x_1^{N,k}, x_2^{N,k})$ *and player 1 accepts. In particular, the unique subgame perfect equilibrium outcome is such that in round 1 player 1 commits to* $c_1 = 0$, *player 2 commits to* $c_2 = x_2^{N,1}$, *player 1 proposes* $(x_1^{N,1}, x_2^{N,1})$ *and player 2 accepts.*

**Proof.** Suppose the bargaining procedure consists of $N$ potential rounds. We prove the statement by induction on the round $k$, starting from the last round $N$ and then working our way backwards.

We first analyze the subgame perfect equilibrium behavior in the last round — round $N$. Let us first assume that $N$ is odd. Hence, player 1 is the proposer at round $N$. Note that the subgame starting at round $N$ consists only of one round, and that the commitment costs incurred before round $N$ are all sunk costs. Therefore, the subgame that starts at round $N$ is essentially identical to the one round bargaining procedure which we analyzed in the previous section.

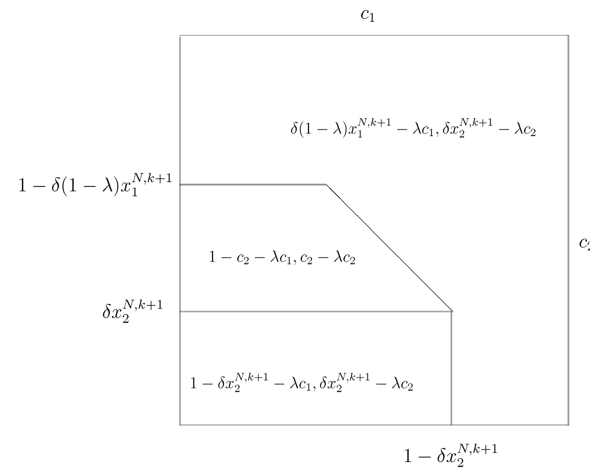


**Fig. 2.** Subgame perfect equilibrium utilities after every pair $(c_1, c_2)$ in round $k$.

By Theorem 1 it then follows that at round $N$ player 1 commits to $c_1 = 0$, player 2 commits to $c_2 = 1 - \delta y_1 = x_2^{N,N}$, player 1 proposes $(\delta y_1, 1 - \delta y_1) = (x_1^{N,N}, x_2^{N,N})$ and player 2 accepts. A similar proof can be given for the case where $N$ is even.

Consider now an earlier round $k$, and assume that the statement in the theorem holds for the next round $k+1$. Let us first assume that $k$ is odd. Hence, player 1 is the proposer at round $k$. Suppose that player 1's proposal at round $k$ would be rejected. Then, by the induction assumption, at round $k+1$ player 2 would commit to $c_2 = 0$, player 1 would commit to $c_1 = x_1^{N,k+1}$, player 2 would propose $(x_1^{N,k+1}, x_2^{N,k+1})$ and player 1 would accept. The corresponding utilities would be $x_1^{N,k+1} - \lambda x_1^{N,k+1} = (1-\lambda)x_1^{N,k+1}$ for player 1 and $x_2^{N,k+1}$ for player 2. Hence, if we are in round $k$, then the discounted utilities resulting from a rejected proposal would be $\delta(1-\lambda)x_1^{N,k+1}$ for player 1, and $\delta x_2^{N,k+1}$ for player 2.

By a similar argument as in the proof of Theorem 1, we can then conclude that the subgame perfect equilibrium utilities after every pair $(c_1, c_2)$ of commitment levels in round $k$ are as given by Fig. 2. The only change compared to the proof in Theorem 1 is that we substitute $(1-\lambda)x_1^{N,k+1}$ for $y_1$, and substitute $x_2^{N,k+1}$ for $y_2$.

In a similar way as in the previous section (the case of one round), it can be shown that for every value of $c_2$, player 1's utility is strictly decreasing in his commitment level $c_1$. Hence, player 1 will choose $c_1 = 0$. But then, in a similar way as in the previous section, it can be shown that player 2's best choice is $c_2 = 1 - \delta(1-\lambda)x_1^{N,k+1} = x_2^{N,k}$. So, the unique subgame perfect equilibrium behavior at round $k$ is such that player 1 commits to $c_1 = 0$, player 2 commits to $c_2 = 1 - \delta(1-\lambda)x_1^{N,k+1} = x_2^{N,k}$, player 1 proposes $\delta(1-\lambda)x_1^{N,k+1} = x_1^{N,k}$ for himself, player 1 proposes $1 - \delta(1-\lambda)x_1^{N,k+1} = x_2^{N,k}$ for player 2, and player 2 accepts. Hence, the statement of the theorem follows for round $k$ when $k$ is odd. A similar proof can be designed for the case where $k$ is even. By induction on $k$, the statement holds for every round $k$, and hence the proof is complete. ■

We will now discuss the result of Theorem 2. In particular, we will compare the unique subgame perfect equilibrium outcome in our setting to that in a setting without commitment.

Contrary to the outcome in the setting without commitment, the outcome in our model is not efficient since player 2 – the responder in round 1 – incurs some costs $\lambda x_2^{N,1}$ due to the strictly positive commitment level $x_2^{N,1}$ he chooses. But there is no delay — an agreement is reached immediately, and the full good is divided between the two players. Hence, the only source for inefficiency is the commitment cost incurred by player 2.

Interestingly, if we let the marginal commitment costs $\lambda$ tend to zero, then the recursive equations above would exactly yield the outcomes for the players in the procedure without commitment, but with the roles of the proposer and responder reversed. That is, what player 1 gets in the model with commitment is exactly what he would have gotten as player 2 – the responder in round 1 – in the model without commitment, and *vice versa.*

Let us now see what happens if the potential number $N$ of rounds becomes very large. If $N$ is odd, then it may be verified that the agreed upon amounts $x_1^{N,1}$ and $x_2^{N,1}$ in round 1 are equal to

$$x_1^{N,1} = \frac{\delta(1-\lambda) + (-1)^N \delta^N (1-\lambda)^N}{1+\delta(1-\lambda)} + (-1)^{N-1}\delta^N(1-\lambda)^{N-1} y_1$$

and

$$x_2^{N,1} = \frac{1 - (-1)^N \delta^N (1-\lambda)^N}{1+\delta(1-\lambda)} - (-1)^{N-1}\delta^N(1-\lambda)^{N-1} y_1.$$

If the number of rounds $N$ becomes very large, then

$$x_1^{N,1} \approx \frac{\delta(1-\lambda)}{1+\delta(1-\lambda)} \quad \text{and} \quad x_2^{N,1} \approx \frac{1}{1+\delta(1-\lambda)}$$

which shows that there is a clear first-mover disadvantage. The same actually holds when $N$ is large and even.

The intuition for this first-mover disadvantage is very similar to that given in the previous section for the case of one round. In every round, the proposer does not really benefit from his possibility of choosing a commitment level, as every offer that will be accepted by the responder could also have been made after having chosen a commitment level of 0. In view of this, the best the proposer can do in every round is to choose a commitment level of 0. The responder, anticipating on this, will be able to extract the full surplus from the proposer by choosing the maximum possible commitment level that the proposer is still willing to meet. As such, it is the responder that has a comparative advantage in every round, and not the proposer. Overall, player 1 — who is the proposer in round 1 – will face a disadvantage compared to player 2.

If $N$ is large, and in addition the marginal commitment costs $\lambda$ would tend to zero, then in the limit we would obtain the reversed Rubinstein outcome

$$x_1^{N,1} \approx \frac{\delta}{1+\delta} \quad \text{and} \quad x_2^{N,1} \approx \frac{1}{1+\delta}.$$

Hence, player 1 – the first mover – gets exactly what he would have gotten as the second mover in the original Rubinstein setting, and player 2 gets precisely what he would have gotten as the first mover in the original Rubinstein setting.

A similar argument as in the end of Section 3 leads to the conclusion that it would be fairly easy to incorporate outside options into the analysis. As shown by Binmore et al. (1989), a player's outside option influences the equilibrium payoffs in a complete information setup if and only if the option yields higher payoff to the player than the deal resulting in the model without outside options (see also Compte and Jehiel, 2002). When choosing their commitments in our game, players must ensure that one's commitment leaves the opponent weakly better off than the opponent's outside option and weakly better off than the current round impasse payoff (induction assumption). If there were both outside options and breakdown payoffs in our model, then the responder's optimal commitment would equal the maximum of the proposer's current-round impasse payoff (continuation payoff) and outside option payoff.

Let us then discuss the relation to endogenous commitment models by Fershtman and Seidmann (1993) and Li (2007) (See also Compte and Jehiel, 2007). In those models, commitments are endogenous as in our model but cannot be freely chosen. In those models, there is a positive association between a proposed share to the opponent at the current round and the opponent's commitment in future rounds. Proposals must not be overly generous since rejecting an overly generous proposal improves the opponent's bargaining position in future rounds to an extent that it allows the opponent to reap an even higher share of the pie than the one currently proposed. In the present model both proposals and commitments are short-lived and do not exhibit such history-dependencies.

Recently, Ellingsen and Miettinen (2014) studied a dynamic version of their negotiations model with strong commitments (Ellingsen and Miettinen, 2008). In the dynamic model the successful commitments have a more irreversible nature than in our paper. The commitments have a stochastic duration and decay of commitments follows a Poisson process.[4] When two uncommitted negotiators meet in their model, the proposer is randomly drawn. In the present model, the proposer in each round is pre-specified and commonly known. Moreover, commitments decay with probability one after each negotiation round. In contrast to the present model, Miettinen and Ellingsen show that both players have an incentive to attempt commitments to force concessions from the opponent. The resulting commitment positions are incompatible and the game turns into a variant of a complete information war of attrition with delay in reaching agreements. The first player whose commitment fails accepts the offer that her opponent is committed to. In our model only the responder at a given round commits and sets the least acceptable offer precisely equal to what the proposer is willing to accept. The first mover has no incentive to commit, since being a first mover already provides commitment power and a capacity to tailor the take-it-or-leave-it offer to precisely match with the responder's commitment. This has the advantage of saving the commitment cost and not risking an impasse and thus a delayed agreement. The differences between our results and those of Ellingsen and Miettinen (2014) are on the one hand due to stronger leverage that the term irreversibility of the commitment provides in forcing greater concessions from an uncommitted opponent. On the other hand, the differences are also due to the fact that in their model both players have an incentive to attempt commitments in order to turn the position of an uncommitted weak responder to a position of a credibly committed proposer.

## 5. Concluding remarks

### 5.1. Commitment costs

In our model we have assumed that the commitment costs for both players are given by $\lambda c$, where $c$ is the amount committed to, and $\lambda$ is some fixed number. In fact, we do not really need this specific functional form for the commitment costs. Instead, we could assume that the commitment costs are given by a more general function $\gamma(c)$, where $\gamma(0) = 0$, the function $\gamma$ is non-decreasing in the commitment level $c$, $\gamma(c) > 0$ for $c > 0$ and $\gamma(1) \leq 1 - \delta$. The reader may verify that under these assumptions, there would also be a unique subgame perfect equilibrium outcome in which the proposer at round 1 faces a first-mover disadvantage. The outcome can be computed by a recursive formula similar to the one used in Theorem 2. Also under these assumptions we would obtain the reversed Rubinstein outcome $(\delta/(1+\delta), 1/(1+\delta))$ if we let the number of rounds go to infinity, and let the commitment costs go to zero. However, in the paper we have chosen the specific functional form $\lambda c$ for the commitment costs as to keep the presentation and the analysis as simple as possible.

[4] See Maskin and Tirole (1988) for a related model of dynamic capacity commitment in a duopoly model. In the capacity competition case, inefficiently large capacity does not result in zero flow-payoff like in the bargaining model of Ellingsen and Miettinen, so efficiency losses are in that sense smaller.

### 5.2. Asymmetric players

In our analysis we have assumed that both players are *symmetric* in the sense that they share the same discount factor $\delta$ and the same marginal cost for commitment $\lambda$. The same analysis could have been carried out for the *asymmetric case*, in which both players hold different discount factors $\delta_1$ and $\delta_2$, and different marginal costs of commitment $\lambda_1$ and $\lambda_2$. We stuck to the symmetric case just for the sake of simplicity.

### 5.3. Alternative to subgame perfect equilibrium

In this paper we have used the concept of subgame perfect equilibrium to analyze the bargaining game. In the working paper version of this paper, Miettinen and Perea (2014), we use the weaker notion of common belief in future rationality (Perea, 2014) to establish the same results. In fact, subgame perfect equilibrium is a strict refinement of common belief in future rationality (see Perea and Predtetchinski, 2014 for a formal proof), and in the working paper version we show that in the bargaining game with commitment there is a unique outcome that is possible under common belief in future rationality, and this outcome is the same as the unique subgame perfect equilibrium outcome we find in this paper. So, the results in the paper do not change if we use the weaker notion of common belief in future rationality rather than subgame perfect equilibrium.

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