# EPICENTER Spring Course in Epistemic Game Theory


**Maastricht University, June 25–July 9, 2018**
**Exam**

July 9, 9.00–12.00, Room G 0.03

Good luck!

**Problem 1: The dancing competition** (35 points)

You and Barbara are the jury members of a dancing competition. After every performance, you and Barbara report an integer grade between 1 and 10, and the final grade for the candidate will just be the average of the two reported grades.

You have just seen the first dancer, and you value her performance at 8, whereas Barbara values the performance at 7. That is, your desired grade is an 8 and Barbara's desired grade is a 7. Since you are both loud and clear about your opinion, you both know the desired grade of the other person. Now it is time for both of you to report a grade, which may well be different from your desired grade, because the final grade also depends on the other person's reported grade.

Suppose that the preferences for you and Barbara are as follows: The closer the final grade is to your desired grade, the higher your utility. But remember that Barbara and you have different desired grades.

**(a)** (4 points) Which choices (reported grades) are rational for you? For every rational choice, find a belief about Barbara's choice for which that choice is optimal. For every irrational choice, find another choice, or randomized choice, that strictly dominates it.

**(b)** (8 points) Which grades can you and Barbara rationally report under common belief in rationality? Which final grades are possible under common belief in rationality? Which algorithm do you use here?

Barbara was shocked by the large difference in grades you both reported for the first dancer, and it has made her rather insecure. Her preferences have changed because of this unpleasant experience. From now on, her objective is to report the same grade as you. That is, if she reports the same grade as you, her utility will be 1, whereas her utility will be 0 if she reports a different grade than you. Your preferences are similar as before: The closer the final grade is to your desired grade, the better.

The second dancer in the competition is Chris, who danced the jive just like John Travolta in Saturday Night Fever. Your desired grade for Chris' marvellous performance is therefore a 9.

**(c)** (5 points) Which grades can you and Barbara rationally report under common belief in rationality? Which final grades are possible under common belief in rationality?

**(d)** (4 points) Make a beliefs diagram with solid arrows that only includes the choices you found in (c). Which of your belief hierarchies in the diagram expresses common belief in rationality? Which of your belief hierarchies in the diagram is simple? Explain your answers.

**(e)** (3 points) Translate this beliefs diagram into an epistemic model.

**(f)** (8 points) Suppose that your utility function is given by

$$u_1 = 4 - 4 \cdot (\text{final grade } - 9)^2.$$

Show that under common belief in rationality with a simple belief hierarchy, there is only one grade you can rationally report. Which one? Please explain. Show formally that the other grades cannot be reported rationally by you under common belief in rationality with a simple belief hierarchy.

**(g)** (3 points) What epistemic condition distinguishes common belief in rationality from Nash equilibrium? How appealing do you find this epistemic condition? Please discuss.

**Problem 2: The last cake** (35 points)

You and Barbara enter the bakery shop at the same time, and see a delicious "Limburgse vlaai" (a local cake with fruits) that you both definitely want to have. However, the baker sadly announces that this is the last cake left. To decide who will get the cake, the baker proposes the following auction between you and Barbara. You both must whisper a price from $\{0, 10, 20, 30, 40, 50\}$ in the baker's ear, and the person who names the highest price will get the cake. The price that must be paid for the cake is the average of the two prices that you and Barbara chose. If you both choose the same price, then the baker will toss a coin to decide who will get the cake.

Suppose you and Barbara both value the cake at 40 euros, and you both know this from each other. The preferences for you and Barbara are as follows: If you win the cake and must pay a price of $p$, then your utility will be $40 - p$. If you do not get the cake, your utility will be 0. The same for Barbara.

**(a)** (5 points) Model this situation as a game between you and Barbara. That is, make a table with the choices and utilities for you and Barbara.

**(b)** (3 points) What prices can you and Barbara rationally choose under common belief in rationality with *standard* beliefs?

We will now turn to cautious reasoning with lexicographic beliefs.

**(c)** (4 points) Explain the difference between primary belief in the opponent's rationality, respecting the opponent's preferences, and assuming the opponent's rationality.

**(d)** (5 points) Which choices are rational for you under a cautious lexicographic belief? For every rational choice, find a cautious lexicographic belief about Barbara's choice for which this choice is optimal. For every irrational choice, find another choice, or randomized choice, that *weakly* dominates it.

**(e)** (8 points) Find the choices you can rationally make under common full belief in "caution and primary belief in rationality". Which algorithm do you use?

**(f)** (3 points) Which choices can you rationally make under common full belief in "caution and respect of preferences"? Which choices can you rationally make under common assumption of rationality?
(Hint: Do not make any new computations. Use the logical relationships between the three concepts.)

**(g)** (7 points) Construct an epistemic model, with one type for you and one type for Barbara, where both types express common full belief in "caution and respect of preferences".

**Problem 3: Summer holiday** (30 points)

You and Barbara are planning your summer holidays, and there is the choice between Rome and Paris. Since you could not reach an agreement on the final destination, both of you will just independently book a flight to either Rome or Paris. If you both happen to fly to Rome, then you must still decide what to do there. Upon arriving in Rome, you can both vote for either a historical tour through the city, or an audience with the pope. If you both vote for the same activity, you will do this activity together. If you both vote for different activities, you will both do the activity of your choice alone.

Similarly, if you both happen to fly to Paris, there are three possible activities that you can vote for upon arriving in Paris: visiting the Eiffel Tower, a night at the Moulin Rouge, or a day at the Louvre museum. Again, if you vote for the same activity you will do it together. Otherwise, you will both do the activity of your choice alone.

The utilities that you and Barbara would derive from the various activities are given by the following table:

| | historical tour | audience with Pope | Eiffel Tower | Moulin Rouge | Louvre | activity together |
|---|---|---|---|---|---|---|
| you | 3 | 5 | 0 | 4 | 0 | utility +2 |
| Barbara | 2 | 2 | 4 | 3 | 4 | utility +2 |

The numbers depict the utilities in case you do the activity alone. If you do the activity together, the utility for you and Barbara will both increase by 2.

It may also happen that you both fly to different cities. Since you have an affair with somebody in Paris, and Barbara has a love affair in Rome, you would not mind flying to Paris alone, and Barbara would be happy to fly to Rome alone. In this case you would both spend a nice week with your secret lovers, and the utilities for Barbara and you would both be 6. On the other hand, if you fly to Rome and Barbara flies to Paris, you will both spend your holidays alone, and you would both have a utility of only 3.

**(a)** (4 points) Explain the difference between belief in the opponent's future rationality and strong belief in the opponent's rationality.

**(b)** (5 points) Model the story above as a dynamic game between you and Barbara, with three different information sets.

**(c)** (8 points) Which strategies can you and Barbara rationally choose under common belief in future rationality? Do you expect to spend the holiday together with Barbara? Which algorithm do you use?

**(d)** (8 points) Which strategies can you and Barbara rationally choose under common strong belief in rationality? Do you expect to spend the holiday together with Barbara? Do you expect to do the activity together? Which algorithm do you use?

**(e)** (5 points) Explain intuitively the reasoning of common belief in future rationality, and common strong belief in rationality, in this particular game. Also explain why both concepts lead to different outcomes.